練習問題 2-A

1. (1) 与式 $= \dfrac{9x^2y^6}{-8x^6y^3}$

$= -\dfrac{\boldsymbol{9y^3}}{\boldsymbol{8x^4}}$

(2) 与式 $= \dfrac{x(x-y)}{(x+y)(x-y)} + \dfrac{y(x+y)}{(x-y)(x+y)} - \dfrac{x^2+y^2}{x^2-y^2}$

$= \dfrac{x^2-xy+xy+y^2-(x^2+y^2)}{(x+y)(x-y)}$

$= \dfrac{0}{(x+y)(x-y)} = \mathbf{0}$

(3) 与式 $= \dfrac{1(x+y)(x+y+z)}{x(x+y)(x+y+z)} - \dfrac{y(x+y+z)}{x(x+y)(x+y+z)} - \dfrac{zx}{x(x+y)(x+y+z)}$

$= \dfrac{(x+y)^2+z(x+y)-y(x+y)-yz-zx}{x(x+y)(x+y+z)}$

$= \dfrac{(x+y)^2+(z-y)(x+y)-z(x+y)}{x(x+y)(x+y+z)}$

$= \dfrac{(x+y)\{(x+y)+(z-y)-z\}}{x(x+y)(x+y+z)}$

$= \dfrac{x(x+y)}{x(x+y)(x+y+z)}$

$= \dfrac{\mathbf{1}}{\boldsymbol{x+y+z}}$

(4) 与式 $= \dfrac{(a+2)(a-3)}{(a+4)(a-3)} \times \dfrac{(a+4)(a-4)}{(a+2)(a-2)} \times \dfrac{a-2}{a-4}$

$= \mathbf{1}$

(5) 与式 $= \dfrac{\left(\dfrac{a^2+1}{a^2-1}-1\right)\times(a^2-1)}{\left(\dfrac{a-1}{a+1}-\dfrac{a+1}{a-1}\right)\times(a^2-1)}$

$= \dfrac{a^2+1-(a^2-1)}{(a-1)^2-(a+1)^2}$

$= \dfrac{2}{-4a} = -\dfrac{\mathbf{1}}{\boldsymbol{2a}}$

2. (1) 与式 $= \dfrac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} + \dfrac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \dfrac{(3+2\sqrt{3}+1)+(3-2\sqrt{3}+1)}{3-1}$

$= \dfrac{8}{2} = \mathbf{4}$

(2) 与式 $= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \mathbf{1}$

(3) 与式 $= (x+y)^2 - 2xy$

$= 4^2 - 2\cdot 1 = \mathbf{14}$

(4) 与式 $= (x+y)^3 - 3xy(x+y)$

$= 4^3 - 3\cdot 4\cdot 1 = \mathbf{52}$

3. (1) 与式 $= \{\sqrt{5}+(\sqrt{3}-\sqrt{2})\}\{\sqrt{5}-(\sqrt{3}-\sqrt{2})\}$

$= (\sqrt{5})^2 - (\sqrt{3}-\sqrt{2})^2$

$= 5-(3-2\sqrt{6}+2) = \boldsymbol{2\sqrt{6}}$

(2) 与式 $= \dfrac{(1-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} + \dfrac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{2-3\sqrt{3}+3}{4-3} - \dfrac{3+2\sqrt{3}+1}{3-1}$

$= 5-3\sqrt{3} - \dfrac{4+2\sqrt{3}}{2}$

$= 5-3\sqrt{3}-(2+\sqrt{3})$

$= \boldsymbol{3-4\sqrt{3}}$

(3) 与式 $= \dfrac{2+\sqrt{3}\,i}{2-\sqrt{3}\,i} + \dfrac{2-\sqrt{3}\,i}{2+\sqrt{3}\,i}$

$= \dfrac{(2+\sqrt{3}\,i)^2}{(2-\sqrt{3}\,i)(2+\sqrt{3}\,i)} + \dfrac{(2-\sqrt{3}\,i)^2}{(2+\sqrt{3}\,i)(2-\sqrt{3}\,i)}$

$= \dfrac{(4+2\sqrt{3}\,i+3\,i^2)+(4-2\sqrt{3}\,i+3\,i^2)}{4-3\,i^2}$

$= \dfrac{(4+2\sqrt{3}\,i-3)+(4-2\sqrt{3}\,i-3)}{4+3}$

$= \dfrac{\mathbf{2}}{\mathbf{7}}$

（4） 与式 $= \dfrac{(\sqrt{2}+i)^2}{(\sqrt{2}-i)(\sqrt{2}+i)}$

$- \dfrac{(\sqrt{2}-i)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$= \dfrac{(2+2\sqrt{2}\,i+i^2)-(2-2\sqrt{2}\,i+i^2)}{2-i^2}$

$= \dfrac{(2+2\sqrt{2}\,i-1)-(2-2\sqrt{2}\,i-1)}{2+1}$

$= \boldsymbol{\dfrac{4\sqrt{2}}{3}\,i}$

4. （1） $\sqrt{5}-3<0,\ -2+\sqrt{5}>0$ なので

与式 $= \dfrac{|\sqrt{5}-3|}{|-2+\sqrt{5}|}$

$= \dfrac{-(\sqrt{5}-3)}{\sqrt{5}-2}$

$= \dfrac{3\sqrt{5}+6-5-2\sqrt{5}}{5-4}$

$= \boldsymbol{1+\sqrt{5}}$

（2） 与式 $= \sqrt{(-2)^2+(\sqrt{2})^2} - \sqrt{(-\sqrt{5})^2+(-1)^2}$

$= \sqrt{6}-\sqrt{6} = \mathbf{0}$

（3） 与式 $= \dfrac{|\sqrt{2}+3\,i|}{|2-\sqrt{3}\,i|}$

$= \dfrac{\sqrt{(\sqrt{2})^2+3^2}}{\sqrt{2^2+(\sqrt{3})^2}}$

$= \dfrac{\sqrt{11}}{\sqrt{7}} = \boldsymbol{\dfrac{\sqrt{77}}{7}}$

練習問題 2-B

1. （1） 与式 $= \dfrac{2a^2}{(2a+b)(2a-b)} - \dfrac{a-b}{2a-b}$

$= \dfrac{2a^2}{(2a+b)(2a-b)} - \dfrac{(a-b)(2a+b)}{(2a-b)(2a+b)}$

$= \dfrac{2a^2-(a-b)(2a+b)}{(2a+b)(2a-b)}$

$= \dfrac{2a^2-(2a^2-ab-b^2)}{(2a+b)(2a-b)}$

$= \dfrac{ab+b^2}{(2a+b)(2a-b)}$

$= \boldsymbol{\dfrac{b(a+b)}{(2a+b)(2a-b)}}$

（2） 与式 $= \dfrac{a+1}{(a-1)(a+1)} - \dfrac{a-1}{(a+1)(a-1)}$

$- \dfrac{2}{a^2+1} - \dfrac{4}{a^4+1}$

$= \dfrac{a+1-(a-1)}{(a-1)(a+1)} - \dfrac{2}{a^2+1} - \dfrac{4}{a^4+1}$

$= \dfrac{2}{a^2-1} - \dfrac{2}{a^2+1} - \dfrac{4}{a^4+1}$

$= \dfrac{2(a^2+1)}{(a^2-1)(a^2+1)}$

$- \dfrac{2(a^2-1)}{(a^2+1)(a^2-1)} - \dfrac{4}{a^4+1}$

$= \dfrac{2(a^2+1)-2(a^2-1)}{(a^2-1)(a^2+1)} - \dfrac{4}{a^4+1}$

$= \dfrac{4}{a^4-1} - \dfrac{4}{a^4+1}$

$= \dfrac{4(a^4+1)}{(a^4-1)(a^4+1)} - \dfrac{4(a^4-1)}{(a^4+1)(a^4-1)}$

$= \dfrac{4(a^4+1)-4(a^4-1)}{(a^4-1)(a^4+1)}$

$= \boldsymbol{\dfrac{8}{a^8-1}}$

（3） 与式 $= \dfrac{x^3}{x+\dfrac{1\cdot x}{\left(x-\dfrac{1}{x}\right)\cdot x}}$

$= \dfrac{x^3}{x+\dfrac{x}{x^2-1}}$

$= \dfrac{x^3(x^2-1)}{\left(x+\dfrac{x}{x^2-1}\right)(x^2-1)}$

$= \dfrac{x^3(x^2-1)}{x(x^2-1)+x}$

$= \dfrac{x^3(x^2-1)}{x^3} = \boldsymbol{x^2-1}$

（4） 与式 $= \dfrac{(x+2)(x+3)}{\left(1-\dfrac{1}{x+3}\right)(x+3)}$

$- \dfrac{(x+2)(x+1)}{\left(1+\dfrac{1}{x+1}\right)(x+1)}$

$= \dfrac{(x+2)(x+3)}{(x+3)-1} - \dfrac{(x+2)(x+1)}{(x+1)+1}$

$= \dfrac{(x+2)(x+3)}{x+2} - \dfrac{(x+2)(x+1)}{x+2}$

$= (x+3)-(x+1) = \mathbf{2}$

（5）与式 $= \dfrac{2a}{\dfrac{1\cdot a}{\left(1-\dfrac{1}{a}\right)\cdot a} - \dfrac{1\cdot a}{\left(1+\dfrac{1}{a}\right)\cdot a}}$

$= \dfrac{2a}{\dfrac{a}{a-1} - \dfrac{a}{a+1}}$

$= \dfrac{2a(a+1)(a-1)}{\left(\dfrac{a}{a-1} - \dfrac{a}{a+1}\right)(a+1)(a-1)}$

$= \dfrac{2a(a+1)(a-1)}{a(a+1)-a(a-1)}$

$= \dfrac{2a(a+1)(a-1)}{2a}$

$= \boldsymbol{(a+1)(a-1)}$

$= \boldsymbol{a^2-1}$

2.（1）与式 $= \dfrac{\sqrt{2}-\sqrt{1}}{(\sqrt{2}+\sqrt{1})(\sqrt{2}-\sqrt{1})}$

$+ \dfrac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$+ \dfrac{\sqrt{4}-\sqrt{3}}{(\sqrt{4}+\sqrt{3})(\sqrt{4}-\sqrt{3})}$

$= \dfrac{\sqrt{2}-1}{2-1} + \dfrac{\sqrt{3}-\sqrt{2}}{3-2} + \dfrac{2-\sqrt{3}}{4-3}$

$= (\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(2-\sqrt{3})$

$= \mathbf{1}$

（2）与式 $= \dfrac{1+\sqrt{2}-\sqrt{3}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}}$

$= \dfrac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2})^2-3}$

$= \dfrac{1+\sqrt{2}-\sqrt{3}}{(1+2\sqrt{2}+2)-3}$

$= \dfrac{1+\sqrt{2}-\sqrt{3}}{2\sqrt{2}}$

$= \dfrac{(1+\sqrt{2}-\sqrt{3})\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}}$

$= \boldsymbol{\dfrac{\sqrt{2}+2-\sqrt{6}}{4}}$

3. $x = 2\sqrt{a-1}$ を $\sqrt{a^2-x^2}$ に代入すると

$\sqrt{a^2-x^2} = \sqrt{a^2-(2\sqrt{a-1})^2}$

$= \sqrt{a^2-4(a-1)}$

$= \sqrt{a^2-4a+4}$

$= \sqrt{(a-2)^2}$

$= |a-2|$

i）$a-2 \geqq 0$，すなわち，$a \geqq 2$ のとき

$|a-2| = a-2$

ii）$a-2<0$，$a \geqq 1$，すなわち，$1 \leqq a < 2$ のとき

$|a-2| = -(a-2) = -a+2$

よって

$$\begin{cases} a \geqq 2 \text{ のとき} & \boldsymbol{a-2} \\ 1 \leqq a < 2 \text{ のとき} & \boldsymbol{-a+2} \end{cases}$$

4. $\alpha = a+b\,i$, $\beta = c+d\,i$ とおく．

（1）左辺 $= \overline{(a+b\,i)+(c+d\,i)}$

$= \overline{(a+c)+(b+d)\,i}$

$= (a+c)-(b+d)\,i$

右辺 $= \overline{(a+b\,i)} + \overline{(c+d\,i)}$

$= (a-b\,i)+(c-d\,i)$

$= (a+c)-(b+d)\,i$

よって，左辺 = 右辺

（2）左辺 $= \overline{(a+b\,i)-(c+d\,i)}$

$= \overline{(a-c)+(b-d)\,i}$

$= (a-c)-(b-d)\,i$

右辺 $= \overline{(a+b\,i)} - \overline{(c+d\,i)}$

$= (a-b\,i)-(c-d\,i)$

$= (a-c)-(b-d)\,i$

よって，左辺 = 右辺

（3）左辺 $= \overline{(a+b\,i)(c+d\,i)}$

$= \overline{ac+ad\,i+bc\,i+bd\,i^2}$

$= \overline{(ac-bd)+(ad+bc)\,i}$

$= (ac-bd)-(ad+bc)\,i$

右辺 $= \overline{(a+b\,i)}\cdot\overline{(c+d\,i)}$

$= (a-b\,i)\cdot(c-d\,i)$

$= ac-ad\,i-bc\,i+bd\,i^2$

$= (ac-bd)-(ad+bc)\,i$

よって，左辺 = 右辺

（4） $\beta \fallingdotseq 0$ なので，$c \fallingdotseq 0$，$d \fallingdotseq 0$ すなわち，$c^2 + d^2 \fallingdotseq 0$ である．

$$\text{左辺} = \overline{\left(\frac{a+b\,i}{c+d\,i}\right)} = \overline{\left(\frac{(a+b\,i)(c-d\,i)}{(c+d\,i)(c-d\,i)}\right)}$$

$$= \overline{\left(\frac{(ac+bd)+(bc-ad)\,i}{c^2+d^2}\right)}$$

$$= \overline{\frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}\ i}$$

$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}\ i$$

$$\text{右辺} = \frac{\overline{a+b\,i}}{\overline{c+d\,i}} = \frac{a-b\,i}{c-d\,i}$$

$$= \frac{(a-b\,i)(c+d\,i)}{(c-d\,i)(c+d\,i)}$$

$$= \frac{(ac+bd)-(ad-bc)\,i}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}\ i$$

よって，左辺 = 右辺

$\blacksquare$