2章　行列

§1　行列（p.50〜p.68）

問1

$\begin{pmatrix} 3 & 4 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$について

(1, 2)成分は，$\mathbf{4}$

(2, 1)成分は，$\mathbf{-1}$

$\begin{pmatrix} 67 & 52 \\ 20 & 87 \end{pmatrix}$について

(1, 2)成分は，$\mathbf{52}$

(2, 1)成分は，$\mathbf{20}$

$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$について

(1, 2)成分は，$\boldsymbol{b}$

(2, 1)成分は，$\boldsymbol{d}$

問2

両辺の対応する成分がすべて等しいので

$$\begin{cases} 2a-3b=-5 & \cdots① \\ c-d=5 & \cdots② \\ a+2b=8 & \cdots③ \\ 3c+5d=-17 & \cdots④ \end{cases}$$

$$\begin{array}{lrl} ① & & 2a-3b=-5 \\ ③\times -2 & +) & -2a-4b=-16 \\ \hline & & -7b=-21 \\ & & b=3 \end{array}$$

これを③に代入すると，

$a+6=8$であるから，$a=2$

$$\begin{array}{lrl} ②\times 5 & & 5c-5d=25 \\ ④ & +) & 3c+5d=-17 \\ \hline & & 8c \quad\ =8 \\ & & c \quad\ =1 \end{array}$$

これを②に代入すると，

$1-d=5$であるから，$d=-4$

以上より，$\boldsymbol{a=2,\ b=3,\ c=1,\ d=-4}$

問3

（1）与式 $=\begin{pmatrix} 2+(-6) & 3+8 \\ -1+7 & -1+0 \end{pmatrix}$

$=\begin{pmatrix} \mathbf{-4} & \mathbf{11} \\ \mathbf{6} & \mathbf{-1} \end{pmatrix}$

（2）与式 $=\begin{pmatrix} 3+(-2) & 7+3 & 3+(-3) \\ -6+8 & 8+3 & 1+2 \end{pmatrix}$

$=\begin{pmatrix} \mathbf{1} & \mathbf{10} & \mathbf{0} \\ \mathbf{2} & \mathbf{11} & \mathbf{3} \end{pmatrix}$

問4

（1）与式 $=\begin{pmatrix} 2 & 6 & -9 \\ -3 & -1 & -3 \end{pmatrix}+\begin{pmatrix} 2 & 7 & 2 \\ -1 & 7 & 3 \end{pmatrix}$

$=\begin{pmatrix} 2+2 & 6+7 & -9+2 \\ -3+(-1) & -1+7 & -3+3 \end{pmatrix}$

$=\begin{pmatrix} \mathbf{4} & \mathbf{13} & \mathbf{-7} \\ \mathbf{-4} & \mathbf{6} & \mathbf{0} \end{pmatrix}$

（2）与式 $=\begin{pmatrix} 2 & 6 & -9 \\ -3 & -1 & -3 \end{pmatrix}+\begin{pmatrix} -2 & 1 & -2 \\ 7 & -1 & 5 \end{pmatrix}+\begin{pmatrix} 2 & 7 & 2 \\ -1 & 7 & 3 \end{pmatrix}$

$=\begin{pmatrix} 2+(-2)+2 & 6+1+7 & -9+(-2)+2 \\ -3+7+(-1) & -1+(-1)+7 & -3+5+3 \end{pmatrix}$

$=\begin{pmatrix} \mathbf{2} & \mathbf{14} & \mathbf{-9} \\ \mathbf{3} & \mathbf{5} & \mathbf{5} \end{pmatrix}$

問5

左辺 $=\begin{pmatrix} x+2 & 2y-9 \\ 1+z & -7+w+3 \end{pmatrix}$

$=\begin{pmatrix} x+2 & 2y-9 \\ 1+z & w-4 \end{pmatrix}$

よって，$\begin{pmatrix} x+2 & 2y-9 \\ 1+z & w-4 \end{pmatrix}=\begin{pmatrix} 10 & y-7 \\ -1 & 1 \end{pmatrix}$

両辺の対応する成分がすべて等しいので

$$\begin{cases} x+2=10 & \cdots① \\ 2y-9=y-7 & \cdots② \\ 1+z=-1 & \cdots③ \\ w-4=1 & \cdots④ \end{cases}$$

①より，$x=8$

②より，$y=2$

③より，$z=-2$

④より，$w=5$

以上より，$\boldsymbol{x=8,\ y=2,\ z=-2,\ w=5}$

問6

（1）与式 $=\begin{pmatrix}-6-(-3) & 1-(-7) & 5-8 \\ 1-8 & -6-(-5) & 8-1\end{pmatrix}$

$=\begin{pmatrix}\boldsymbol{-3} & \boldsymbol{8} & \boldsymbol{-3} \\ \boldsymbol{-7} & \boldsymbol{-1} & \boldsymbol{7}\end{pmatrix}$

（2）与式 $=\begin{pmatrix}-2-0 & 4-(-6) \\ -1-(-9) & 6-4 \\ 3-(-4) & 6-0\end{pmatrix}$

$=\begin{pmatrix}\boldsymbol{-2} & \boldsymbol{10} \\ \boldsymbol{8} & \boldsymbol{2} \\ \boldsymbol{7} & \boldsymbol{6}\end{pmatrix}$

問7

（1）与式 $=\begin{pmatrix}-3 & 2 \\ -2 & 4\end{pmatrix}+\begin{pmatrix}3 & -1 \\ 0 & 4\end{pmatrix}-\begin{pmatrix}-2 & 1 \\ -1 & 2\end{pmatrix}$

$=\begin{pmatrix}-3+3-(-2) & 2+(-1)-1 \\ -2+0-(-1) & 4+4-2\end{pmatrix}$

$=\begin{pmatrix}\boldsymbol{2} & \boldsymbol{0} \\ \boldsymbol{-1} & \boldsymbol{6}\end{pmatrix}$

（2）与式 $=\begin{pmatrix}-3 & 2 \\ -2 & 4\end{pmatrix}-\begin{pmatrix}3 & -1 \\ 0 & 4\end{pmatrix}-\begin{pmatrix}-2 & 1 \\ -1 & 2\end{pmatrix}$

$=\begin{pmatrix}-3-3-(-2) & 2-(-1)-1 \\ -2-0-(-1) & 4-4-2\end{pmatrix}$

$=\begin{pmatrix}\boldsymbol{-4} & \boldsymbol{2} \\ \boldsymbol{-1} & \boldsymbol{-2}\end{pmatrix}$

（3）与式 $=\begin{pmatrix}-3 & 2 \\ -2 & 4\end{pmatrix}-\begin{pmatrix}3 & -1 \\ 0 & 4\end{pmatrix}+\begin{pmatrix}-2 & 1 \\ -1 & 2\end{pmatrix}$

$=\begin{pmatrix}-3-3+(-2) & 2-(-1)+1 \\ -2-0+(-1) & 4-4+2\end{pmatrix}$

$=\begin{pmatrix}\boldsymbol{-8} & \boldsymbol{4} \\ \boldsymbol{-3} & \boldsymbol{2}\end{pmatrix}$

問8

$A=\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}$，$B=\begin{pmatrix}b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23}\end{pmatrix}$とする．

（Ⅰ）

左辺 $=k\left\{\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}\pm\begin{pmatrix}b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23}\end{pmatrix}\right\}$

$=k\begin{pmatrix}a_{11}\pm b_{11} & a_{12}\pm b_{12} & a_{13}\pm b_{13} \\ a_{21}\pm b_{21} & a_{22}\pm b_{22} & a_{23}\pm b_{23}\end{pmatrix}$

$=\begin{pmatrix}k(a_{11}\pm b_{11}) & k(a_{12}\pm b_{12}) & k(a_{13}\pm b_{13}) \\ k(a_{21}\pm b_{21}) & k(a_{22}\pm b_{22}) & k(a_{23}\pm b_{23})\end{pmatrix}$

右辺 $=k\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}\pm k\begin{pmatrix}b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23}\end{pmatrix}$

$=\begin{pmatrix}ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23}\end{pmatrix}\pm\begin{pmatrix}kb_{11} & kb_{12} & kb_{13} \\ kb_{21} & kb_{22} & kb_{23}\end{pmatrix}$

$=\begin{pmatrix}ka_{11}\pm kb_{11} & ka_{12}\pm kb_{12} & ka_{13}\pm kb_{13} \\ ka_{21}\pm kb_{21} & ka_{22}\pm kb_{22} & ka_{23}\pm kb_{23}\end{pmatrix}$

$=\begin{pmatrix}k(a_{11}\pm b_{11}) & k(a_{12}\pm b_{12}) & k(a_{13}\pm b_{13}) \\ k(a_{21}\pm b_{21}) & k(a_{22}\pm b_{22}) & k(a_{23}\pm b_{23})\end{pmatrix}$

よって，左辺 $=$ 右辺

(II)

左辺 $=(k\pm l)\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}$

$=\begin{pmatrix}(k\pm l)a_{11} & (k\pm l)a_{12} & (k\pm l)a_{13} \\ (k\pm l)a_{21} & (k\pm l)a_{22} & (k\pm l)a_{23}\end{pmatrix}$

右辺 $=k\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}\pm l\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}$

$=\begin{pmatrix}ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23}\end{pmatrix}\pm\begin{pmatrix}la_{11} & la_{12} & la_{13} \\ la_{21} & la_{22} & la_{23}\end{pmatrix}$

$=\begin{pmatrix}ka_{11}\pm la_{11} & ka_{12}\pm la_{12} & ka_{13}\pm la_{13} \\ ka_{21}\pm la_{21} & ka_{22}\pm la_{22} & ka_{23}\pm la_{23}\end{pmatrix}$

$=\begin{pmatrix}(k\pm l)a_{11} & (k\pm l)a_{12} & (k\pm l)a_{13} \\ (k\pm l)a_{21} & (k\pm l)a_{22} & (k\pm l)a_{23}\end{pmatrix}$

よって，左辺 $=$ 右辺

(III)

左辺 $=(kl)\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}$

$=\begin{pmatrix}(kl)a_{11} & (kl)a_{12} & (kl)a_{13} \\ (kl)a_{21} & (kl)a_{22} & (kl)a_{23}\end{pmatrix}$

$=\begin{pmatrix}kla_{11} & kla_{12} & kla_{13} \\ kla_{21} & kla_{22} & kla_{23}\end{pmatrix}$

右辺 $=k\left\{l\begin{pmatrix}a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{pmatrix}\right\}$

$=k\begin{pmatrix}la_{11} & la_{12} & la_{13} \\ la_{21} & la_{22} & la_{23}\end{pmatrix}$

$=\begin{pmatrix}kla_{11} & kla_{12} & kla_{13} \\ kla_{21} & kla_{22} & kla_{23}\end{pmatrix}$

よって，左辺 $=$ 右辺

問 9

（1）与式 $=\begin{pmatrix}2&6&6\\3&-4&1\end{pmatrix}+3\begin{pmatrix}6&-1&-3\\-5&2&0\end{pmatrix}$

$=\begin{pmatrix}2&6&6\\3&-4&1\end{pmatrix}+\begin{pmatrix}18&-3&-9\\-15&6&0\end{pmatrix}$

$=\begin{pmatrix}2+18&6+(-3)&6+(-9)\\3+(-15)&-4+6&1+0\end{pmatrix}$

$=\begin{pmatrix}\mathbf{20}&\mathbf{3}&\mathbf{-3}\\\mathbf{-12}&\mathbf{2}&\mathbf{1}\end{pmatrix}$

（2）与式 $=3\begin{pmatrix}2&6&6\\3&-4&1\end{pmatrix}-4\begin{pmatrix}6&-1&-3\\-5&2&0\end{pmatrix}$

$=\begin{pmatrix}6&18&18\\9&-12&3\end{pmatrix}-\begin{pmatrix}24&-4&-12\\-20&8&0\end{pmatrix}$

$=\begin{pmatrix}6-24&18-(-4)&18-(-12)\\9-(-20)&-12-8&3-0\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-18}&\mathbf{22}&\mathbf{30}\\\mathbf{29}&\mathbf{-20}&\mathbf{3}\end{pmatrix}$

（3）与式 $=A-2B+2A+B$

$=3A-B$

$=3\begin{pmatrix}2&6&6\\3&-4&1\end{pmatrix}-\begin{pmatrix}6&-1&-3\\-5&2&0\end{pmatrix}$

$=\begin{pmatrix}6&18&18\\9&-12&3\end{pmatrix}-\begin{pmatrix}6&-1&-3\\-5&2&0\end{pmatrix}$

$=\begin{pmatrix}6-6&18-(-1)&18-(-3)\\9-(-5)&-12-2&3-0\end{pmatrix}$

$=\begin{pmatrix}\mathbf{0}&\mathbf{19}&\mathbf{21}\\\mathbf{14}&\mathbf{-14}&\mathbf{3}\end{pmatrix}$

（4）与式 $=B-A-3A+B$

$=-4A+2B$

$=-4\begin{pmatrix}2&6&6\\3&-4&1\end{pmatrix}+2\begin{pmatrix}6&-1&-3\\-5&2&0\end{pmatrix}$

$=\begin{pmatrix}-8&-24&-24\\-12&16&-4\end{pmatrix}+\begin{pmatrix}12&-2&-6\\-10&4&0\end{pmatrix}$

$=\begin{pmatrix}-8+12&-24+(-2)&-24+(-6)\\-12+(-10)&16+4&-4+0\end{pmatrix}$

$=\begin{pmatrix}\mathbf{4}&\mathbf{-26}&\mathbf{-30}\\\mathbf{-22}&\mathbf{20}&\mathbf{-4}\end{pmatrix}$

問 10

$2A+3X=5B$ より

$3X=-2A+5B$

$X=-\frac{2}{3}A+\frac{5}{3}B$

$=\frac{1}{3}\left\{-2\begin{pmatrix}-2&-2&2\\1&3&-1\\3&1&2\end{pmatrix}+5\begin{pmatrix}3&0&2\\4&3&-1\\3&0&3\end{pmatrix}\right\}$

$=\frac{1}{3}\left\{\begin{pmatrix}4&4&-4\\-2&-6&2\\-6&-2&-4\end{pmatrix}+\begin{pmatrix}15&0&10\\20&15&-5\\15&0&15\end{pmatrix}\right\}$

$=\frac{1}{3}\begin{pmatrix}4+15&4+0&-4+10\\-2+20&-6+15&2+(-5)\\-6+15&-2+0&-4+15\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}19&4&6\\18&9&-3\\9&-2&11\end{pmatrix}$

$=\begin{pmatrix}\mathbf{\frac{19}{3}}&\mathbf{\frac{4}{3}}&\mathbf{2}\\\mathbf{6}&\mathbf{3}&\mathbf{-1}\\\mathbf{3}&\mathbf{-\frac{2}{3}}&\mathbf{\frac{11}{3}}\end{pmatrix}$

問 11

（1）与式 $=\begin{pmatrix}3\cdot3+1\cdot2&3\cdot1+1\cdot5\\4\cdot3+(-2)\cdot2&4\cdot1+(-2)\cdot5\end{pmatrix}$

$=\begin{pmatrix}9+2&3+5\\12-4&4-10\end{pmatrix}$

$=\begin{pmatrix}\mathbf{11}&\mathbf{8}\\\mathbf{8}&\mathbf{-6}\end{pmatrix}$

（2）与式 $=\begin{pmatrix}3\cdot(-2)+2\cdot2\\-1\cdot(-2)+4\cdot2\end{pmatrix}$

$=\begin{pmatrix}-6+4\\2+8\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-2}\\\mathbf{10}\end{pmatrix}$

（3）与式 $=\begin{pmatrix}5\cdot1+(-1)\cdot(-2)\end{pmatrix}$

$=(5+2)$

$=(7)=\mathbf{7}$

（4）与式 $=\begin{pmatrix}1\cdot2+1\cdot0&1\cdot1+1\cdot5&1\cdot3+1\cdot0\\5\cdot2+0\cdot0&5\cdot1+0\cdot5&5\cdot3+0\cdot0\\1\cdot2+4\cdot0&1\cdot1+4\cdot5&1\cdot3+4\cdot0\end{pmatrix}$

$=\begin{pmatrix}2+0&1+5&3+0\\10+0&5+0&15+0\\2&1+20&3+0\end{pmatrix}$

$=\begin{pmatrix}\mathbf{2}&\mathbf{6}&\mathbf{3}\\\mathbf{10}&\mathbf{5}&\mathbf{15}\\\mathbf{2}&\mathbf{21}&\mathbf{3}\end{pmatrix}$

（5）与式 $=\begin{pmatrix}2\cdot1+1\cdot5+3\cdot1&2\cdot1+1\cdot0+3\cdot4\\0\cdot1+5\cdot5+0\cdot1&0\cdot1+5\cdot0+0\cdot4\end{pmatrix}$

$=\begin{pmatrix}2+5+3&2+0+12\\0+25+0&0+0+0\end{pmatrix}$

$=\begin{pmatrix}\mathbf{10}&\mathbf{14}\\\mathbf{25}&\mathbf{0}\end{pmatrix}$

（6）与式 $=\begin{pmatrix} 3\cdot 4 & 3\cdot 0 & 3\cdot 5 \\ (-2)\cdot 4 & (-2)\cdot 0 & (-2)\cdot 5 \\ 1\cdot 4 & 1\cdot 0 & 1\cdot 5 \end{pmatrix}$

$$=\begin{pmatrix} \mathbf{12} & \mathbf{0} & \mathbf{15} \\ \mathbf{-8} & \mathbf{0} & \mathbf{-10} \\ \mathbf{4} & \mathbf{0} & \mathbf{5} \end{pmatrix}$$

問 12

$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$, $B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, $C=\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$とする.

（Ⅰ）

$$k(AB)=k\left\{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right\}$$

$$=k\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} \end{pmatrix}$$

$$=\begin{pmatrix} k(a_{11}b_{11}+a_{12}b_{21}) & k(a_{11}b_{12}+a_{12}b_{22}) \\ k(a_{21}b_{11}+a_{22}b_{21}) & k(a_{21}b_{12}+a_{22}b_{22}) \\ k(a_{31}b_{11}+a_{32}b_{21}) & k(a_{31}b_{12}+a_{32}b_{22}) \end{pmatrix}$$

$$(kA)B=\left\{k\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\right\}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$=\begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \\ ka_{31} & ka_{32} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$=\begin{pmatrix} ka_{11}b_{11}+ka_{12}b_{21} & ka_{11}b_{12}+ka_{12}b_{22} \\ ka_{21}b_{11}+ka_{22}b_{21} & ka_{21}b_{12}+ka_{22}b_{22} \\ ka_{31}b_{11}+ka_{32}b_{21} & ka_{31}b_{12}+ka_{32}b_{22} \end{pmatrix}$$

$$=\begin{pmatrix} k(a_{11}b_{11}+a_{12}b_{21}) & k(a_{11}b_{12}+a_{12}b_{22}) \\ k(a_{21}b_{11}+a_{22}b_{21}) & k(a_{21}b_{12}+a_{22}b_{22}) \\ k(a_{31}b_{11}+a_{32}b_{21}) & k(a_{31}b_{12}+a_{32}b_{22}) \end{pmatrix}$$

$$A(kB)=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\left\{k\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right\}$$

$$=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} kb_{11} & kb_{12} \\ kb_{21} & kb_{22} \end{pmatrix}$$

$$=\begin{pmatrix} a_{11}\cdot kb_{11}+a_{12}\cdot kb_{21} & a_{11}\cdot kb_{12}+a_{12}\cdot kb_{22} \\ a_{21}\cdot kb_{11}+a_{22}\cdot kb_{21} & a_{21}\cdot kb_{12}+a_{22}\cdot kb_{22} \\ a_{31}\cdot kb_{11}+a_{32}\cdot kb_{21} & a_{31}\cdot kb_{12}+a_{32}\cdot kb_{22} \end{pmatrix}$$

$$=\begin{pmatrix} ka_{11}b_{11}+ka_{12}b_{21} & ka_{11}b_{12}+ka_{12}b_{22} \\ ka_{21}b_{11}+ka_{22}b_{21} & ka_{21}b_{12}+ka_{22}b_{22} \\ ka_{31}b_{11}+ka_{32}b_{21} & ka_{31}b_{12}+ka_{32}b_{22} \end{pmatrix}$$

$$=\begin{pmatrix} k(a_{11}b_{11}+a_{12}b_{21}) & k(a_{11}b_{12}+a_{12}b_{22}) \\ k(a_{21}b_{11}+a_{22}b_{21}) & k(a_{21}b_{12}+a_{22}b_{22}) \\ k(a_{31}b_{11}+a_{32}b_{21}) & k(a_{31}b_{12}+a_{32}b_{22}) \end{pmatrix}$$

したがって，$k(AB)=(kA)B=A(kB)$

（Ⅲ）第 1 式

$$左辺=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\left\{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}+\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\right\}$$

$$=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} b_{11}+c_{11} & b_{12}+c_{12} \\ b_{21}+c_{21} & b_{22}+c_{22} \end{pmatrix}$$

$$=\begin{pmatrix} a_{11}(b_{11}+c_{11})+a_{12}(b_{21}+c_{21}) & a_{11}(b_{12}+c_{12})+a_{12}(b_{22}+c_{22}) \\ a_{21}(b_{11}+c_{11})+a_{22}(b_{21}+c_{21}) & a_{21}(b_{12}+c_{12})+a_{22}(b_{22}+c_{22}) \\ a_{21}(b_{11}+c_{11})+a_{32}(b_{21}+c_{21}) & a_{31}(b_{12}+c_{12})+a_{32}(b_{22}+c_{22}) \end{pmatrix}$$

$$右辺=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}+\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

$$=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} \end{pmatrix}+\begin{pmatrix} a_{11}c_{11}+a_{12}c_{21} & a_{11}c_{12}+a_{12}c_{22} \\ a_{21}c_{11}+a_{22}c_{21} & a_{21}c_{12}+a_{22}c_{22} \\ a_{31}c_{11}+a_{32}c_{21} & a_{31}c_{12}+a_{32}c_{22} \end{pmatrix}$$

$$=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{11}c_{11}+a_{12}c_{21} & a_{11}b_{12}+a_{12}b_{22}+a_{11}c_{12}+a_{12}c_{22} \\ a_{21}b_{11}+a_{22}b_{21}+a_{21}c_{11}+a_{22}c_{21} & a_{21}b_{12}+a_{22}b_{22}+a_{21}c_{12}+a_{22}c_{22} \\ a_{31}b_{11}+a_{32}b_{21}+a_{31}c_{11}+a_{32}c_{21} & a_{31}b_{12}+a_{32}b_{22}+a_{31}c_{12}+a_{32}c_{22} \end{pmatrix}$$

$$=\begin{pmatrix} a_{11}(b_{11}+c_{11})+a_{12}(b_{21}+c_{21}) & a_{11}(b_{12}+c_{12})+a_{12}(b_{22}+c_{22}) \\ a_{21}(b_{11}+c_{11})+a_{22}(b_{21}+c_{21}) & a_{21}(b_{12}+c_{12})+a_{22}(b_{22}+c_{22}) \\ a_{21}(b_{11}+c_{11})+a_{32}(b_{21}+c_{21}) & a_{31}(b_{12}+c_{12})+a_{32}(b_{22}+c_{22}) \end{pmatrix}$$

よって，左辺 = 右辺

問 13

（1）$J^2=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$=\begin{pmatrix} 1\cdot 1+0\cdot 0 & 1\cdot 0+0\cdot(-1) \\ 0\cdot 1+(-1)\cdot 0 & 0\cdot 0+(-1)\cdot(-1) \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=E$$

$$K^2=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 0\cdot 0+1\cdot 1 & 0\cdot 1+1\cdot 0 \\ 1\cdot 0+0\cdot 1 & 1\cdot 1+0\cdot 0 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=E$$

$$-L^2=-\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$=-\begin{pmatrix} 0\cdot 0+(-1)\cdot 1 & 0\cdot(-1)+(-1)\cdot 0 \\ 1\cdot 0+0\cdot 1 & 1\cdot(-1)+0\cdot 0 \end{pmatrix}$$

$$=-\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=E$$

よって，$J^2=K^2=-L^2=E$

（2）$LJ=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$=\begin{pmatrix} 0\cdot 1+(-1)\cdot 0 & 0\cdot 0+(-1)\cdot(-1) \\ 1\cdot 1+0\cdot 0 & 1\cdot 0+0\cdot(-1) \end{pmatrix}$$

$$=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}=K$$

$$-JL = -\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 1\cdot 0 + 0\cdot 1 & 1\cdot(-1) + 0\cdot 0 \\ 0\cdot 0 + (-1)\cdot 1 & 0\cdot(-1) + (-1)\cdot 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = K$$

よって，$LJ = -JL = K$

（３）$KJ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 0\cdot 1 + 1\cdot 0 & 0\cdot 0 + 1\cdot(-1) \\ 1\cdot 1 + 0\cdot 0 & 1\cdot 0 + 0\cdot(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = L$$

$$-JK = -\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 1\cdot 0 + 0\cdot 1 & 1\cdot 1 + 0\cdot 0 \\ 0\cdot 0 + (-1)\cdot 1 & 0\cdot 1 + (-1)\cdot 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = L$$

よって，$KJ = -JK = L$

（４）$KL = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0\cdot 0 + 1\cdot 1 & 0\cdot(-1) + 1\cdot 0 \\ 1\cdot 0 + 0\cdot 1 & 1\cdot(-1) + 0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = J$$

$$-LK = -\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 0\cdot 0 + (-1)\cdot 1 & 0\cdot 1 + (-1)\cdot 0 \\ 1\cdot 0 + 0\cdot 1 & 1\cdot 1 + 0\cdot 0 \end{pmatrix}$$

$$= -\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = J$$

よって，$KL = -LK = J$

問 14

（１）与式 $= \begin{pmatrix} 2 & 4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & -2 \end{pmatrix} - \begin{pmatrix} 4 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 2\cdot 2 + 4\cdot 3 & 2\cdot 4 + 4\cdot(-2) \\ 3\cdot 2 + (-2)\cdot 3 & 3\cdot 4 + (-2)\cdot(-2) \end{pmatrix}$$

$$- \begin{pmatrix} 4\cdot 4 + 1\cdot 0 & 4\cdot 1 + 1\cdot(-1) \\ 0\cdot 4 + (-1)\cdot 0 & 0\cdot 1 + (-1)\cdot(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} - \begin{pmatrix} 16 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{-3} \\ \mathbf{0} & \mathbf{15} \end{pmatrix}$$

（２）与式 $= \left\{\begin{pmatrix} 2 & 4 \\ 3 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 0 & -1 \end{pmatrix}\right\}\left\{\begin{pmatrix} 2 & 4 \\ 3 & -2 \end{pmatrix} - \begin{pmatrix} 4 & 1 \\ 0 & -1 \end{pmatrix}\right\}$

$$= \begin{pmatrix} 6 & 5 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6\cdot(-2) + 5\cdot 3 & 6\cdot 3 + 5\cdot(-1) \\ 3\cdot(-2) + (-3)\cdot 3 & 3\cdot 3 + (-3)\cdot(-1) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{3} & \mathbf{13} \\ \mathbf{-15} & \mathbf{12} \end{pmatrix}$$

問 15

$$A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0\cdot 0 + 1\cdot 0 & 0\cdot 1 + 1\cdot 0 \\ 0\cdot 0 + 0\cdot 0 & 0\cdot 1 + 0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$$

$$B^2 = \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0\cdot 0 + 3\cdot 0 & 0\cdot 3 + 3\cdot 0 \\ 0\cdot 0 + 0\cdot 0 & 0\cdot 3 + 0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$$

$$AB = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0\cdot 0 + 1\cdot 0 & 0\cdot 3 + 1\cdot 0 \\ 0\cdot 0 + 0\cdot 0 & 0\cdot 3 + 0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$$

よって，$A^2 = B^2 = AB = O$

問 16

$$AB = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2\cdot 1+1\cdot 0 & 2\cdot 0+1\cdot 3 \\ 4\cdot 1+2\cdot 0 & 4\cdot 0+2\cdot 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$$

$$AC = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2\cdot 0+1\cdot 2 & 2\cdot 0+1\cdot 3 \\ 4\cdot 0+2\cdot 2 & 4\cdot 0+2\cdot 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$$

よって，$AB = AC$，$A \neq 0$であっても，

$B = C$とは限らない．

問 17

$$A^2 = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a\cdot a+b\cdot c & a\cdot b+b\cdot 0 \\ c\cdot a+0\cdot c & c\cdot b+0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} a^2+bc & ab \\ ca & bc \end{pmatrix}$$

よって，$A^2 = O$となるための条件は

$$\begin{cases} a^2+bc=0 & \cdot\cdot\cdot ① \\ ab=0 & \cdot\cdot\cdot ② \\ ca=0 & \cdot\cdot\cdot ③ \\ bc=0 & \cdot\cdot\cdot ④ \end{cases}$$

④を①に代入すると，$a^2 = 0$より，$a = 0$

$a = 0$のとき，②，③は任意のb，cについて

成り立つので，求める条件は，$\boldsymbol{a = 0}$**かつ**$\boldsymbol{bc = 0}$

問 18

$$\boldsymbol{{}^tA = \begin{pmatrix} 2 & 5 \\ -3 & 4 \\ -6 & -1 \end{pmatrix}}, \quad \boldsymbol{{}^tB = \begin{pmatrix} 3 & 4 & -1 \\ -6 & 1 & -6 \\ -5 & 0 & 0 \end{pmatrix}}$$

$$\boldsymbol{{}^tC = \begin{pmatrix} 0 & -6 & -2 \\ 6 & 0 & 5 \\ 2 & -5 & 0 \end{pmatrix}}, \quad \boldsymbol{{}^tD = \begin{pmatrix} 1 & -4 & 5 \end{pmatrix}}$$

$$\boldsymbol{{}^tE = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}, \quad \boldsymbol{{}^tF = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}}$$

問 19

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$，$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$とする．

（Ⅰ）

$$左辺 = {}^t\left\{{}^t\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}\right\}$$

$$= {}^t\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = A = 左辺$$

(II)

$$左辺 = {}^t\left\{k\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}\right\}$$

$$= {}^t\begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{pmatrix}$$

$$= \begin{pmatrix} ka_{11} & ka_{21} \\ ka_{12} & ka_{22} \\ ka_{13} & ka_{23} \end{pmatrix}$$

$$右辺 = k\left\{{}^t\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}\right\}$$

$$= k\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix}$$

$$= \begin{pmatrix} ka_{11} & ka_{21} \\ ka_{12} & ka_{22} \\ ka_{13} & ka_{23} \end{pmatrix}$$

よって，左辺＝右辺

(III)

$$左辺 = {}^t\left\{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}\right\}$$

$$= {}^t\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}+b_{11} & a_{21}+b_{21} \\ a_{12}+b_{12} & a_{22}+b_{22} \\ a_{13}+b_{13} & a_{23}+b_{23} \end{pmatrix}$$

$$右辺 = {}^t\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} + {}^t\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}+b_{11} & a_{21}+b_{21} \\ a_{12}+b_{12} & a_{22}+b_{22} \\ a_{13}+b_{13} & a_{23}+b_{23} \end{pmatrix}$$

よって，左辺＝右辺

問 20

$$AB = \begin{pmatrix} 4 & -2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4\cdot(-2)+(-2)\cdot 1 & 4\cdot 3+(-2)\cdot 4 \\ 0\cdot(-2)+3\cdot 1 & 0\cdot 3+3\cdot 4 \end{pmatrix}$$

$= \begin{pmatrix} -10 & 4 \\ 3 & 12 \end{pmatrix}$

$BA = \begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} -2\cdot 4 + 3\cdot 0 & -2\cdot(-2) + 3\cdot 3 \\ 1\cdot 4 + 4\cdot 0 & 1\cdot(-2) + 4\cdot 3 \end{pmatrix}$

$= \begin{pmatrix} -8 & 13 \\ 4 & 10 \end{pmatrix}$

よって

${}^t(AB) = {}^t\begin{pmatrix} -10 & 4 \\ 3 & 12 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-10} & \mathbf{3} \\ \mathbf{4} & \mathbf{12} \end{pmatrix}$

${}^t(BA) = {}^t\begin{pmatrix} -8 & 13 \\ 4 & 10 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-8} & \mathbf{4} \\ \mathbf{13} & \mathbf{10} \end{pmatrix}$

${}^tA\,{}^tB = {}^t\begin{pmatrix} 4 & -2 \\ 0 & 3 \end{pmatrix}{}^t\begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 4\cdot(-2) + 0\cdot 3 & 4\cdot 1 + 0\cdot 4 \\ -2\cdot(-2) + 3\cdot 3 & -2\cdot 1 + 3\cdot 4 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-8} & \mathbf{4} \\ \mathbf{13} & \mathbf{10} \end{pmatrix}$

${}^tB\,{}^tA = {}^t\begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}{}^t\begin{pmatrix} 4 & -2 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} -2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix}$

$= \begin{pmatrix} -2\cdot 4 + 1\cdot(-2) & -2\cdot 0 + 1\cdot 3 \\ 3\cdot 4 + 4\cdot(-2) & 3\cdot 0 + 4\cdot 3 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-10} & \mathbf{3} \\ \mathbf{4} & \mathbf{12} \end{pmatrix}$

問 21

${}^tA = {}^t\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

（１）Aが対称行列であるための条件は，${}^tA = A$

すなわち，$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$であるから

$\begin{cases} a = a\cdot\cdot\cdot ① \\ c = b\cdot\cdot\cdot ② \\ b = c\cdot\cdot\cdot ③ \\ d = d\cdot\cdot\cdot ④ \end{cases}$

①，④は常に成り立つので，求める条件は，$\boldsymbol{b = c}$

（２）Aが交代行列であるための条件は，${}^tA = -A$

すなわち，$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = -\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$であるから

$\begin{cases} a = -a\cdot\cdot\cdot ① \\ c = -b\cdot\cdot\cdot ② \\ b = -c\cdot\cdot\cdot ③ \\ d = -d\cdot\cdot\cdot ④ \end{cases}$

①，④より，$a = d = 0$

よって，求める条件は，$\boldsymbol{a = d = 0}$，$\boldsymbol{b = -c}$

問 22

（１）A，Bが対称行列であるから，${}^tA = A$，${}^tB = B$

よって

${}^t(kA + lB) = {}^t(kA) + {}^t(lB)$

$= k\,{}^tA + l\,{}^tB$

$= kA + lB$

したがって，${}^t(kA + lB) = kA + lB$であるから，

$kA + lB$は対称行列である．

（２）A，Bが対称行列であるから，${}^tA = -A$，${}^tB = -B$

よって

${}^t(kA + lB) = {}^t(kA) + {}^t(lB)$

$= k\,{}^tA + l\,{}^tB$

$= k(-A) + l(-B)$

$= -kA - lB = -(kA + lB)$

したがって，${}^t(kA + lB) = -(kA + lB)$であるから，

$kA + lB$は交代行列である．

問 23

（１）$2\cdot 4 - (-3)\cdot(-1) = 8 - 3 = 5 \neq 0$より，**正則．**

逆行列は，$\frac{1}{5}\begin{pmatrix} 4 & -(-3) \\ -(-1) & 2 \end{pmatrix} = \boldsymbol{\frac{1}{5}\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}}$

（２）$2\cdot 2 - 1\cdot 4 = 0 - 0 = 0$より，**正則でない．**

（3）$1\cdot1-0\cdot0=1\neq0$より，**正則.**

逆行列は，$\frac{1}{1}\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{0}&\mathbf{1}\end{pmatrix}$

問 24

$4\cdot3-5\cdot2=12-10=2\neq0$より，$A$は正則.

$A^{-1}=\frac{1}{2}\begin{pmatrix}3&-5\\-2&4\end{pmatrix}$

（1）$AX=B$の両辺に左からA^{-1}をかけると

$A^{-1}AX=A^{-1}B$

$EX=A^{-1}B$

$$X=\frac{1}{2}\begin{pmatrix}3&-5\\-2&4\end{pmatrix}\begin{pmatrix}-1&-5\\1&-7\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}-3-5&-15+35\\2+4&10-28\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}-8&20\\6&-18\end{pmatrix}$$

$$=\begin{pmatrix}\mathbf{-4}&\mathbf{10}\\\mathbf{3}&\mathbf{-9}\end{pmatrix}$$

（1）$YA=B$の両辺に右からA^{-1}をかけると

$YAA^{-1}=BA^{-1}$

$YE=BA^{-1}$

$$Y=\begin{pmatrix}-1&-5\\1&-7\end{pmatrix}\left\{\frac{1}{2}\begin{pmatrix}3&-5\\-2&4\end{pmatrix}\right\}$$

$$=\frac{1}{2}\begin{pmatrix}-1&-5\\1&-7\end{pmatrix}\begin{pmatrix}3&-5\\-2&4\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}-3+10&5-20\\3+14&-5-28\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}7&-15\\17&-33\end{pmatrix}$$

$$=\begin{pmatrix}\mathbf{\frac{7}{2}}&-\mathbf{\frac{15}{2}}\\\mathbf{\frac{17}{2}}&-\mathbf{\frac{33}{2}}\end{pmatrix}$$

問 25

$$AB=\begin{pmatrix}1&1\\2&3\end{pmatrix}\begin{pmatrix}5&2\\3&1\end{pmatrix}$$

$$=\begin{pmatrix}5+3&2+1\\10+9&4+3\end{pmatrix}$$

$$=\begin{pmatrix}8&3\\19&7\end{pmatrix}$$

$$A^{-1}=\frac{1}{1\cdot3-1\cdot2}\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$$

$$=\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$$

$$B^{-1}=\frac{1}{5\cdot1-2\cdot3}\begin{pmatrix}1&-2\\-3&5\end{pmatrix}$$

$$=-\begin{pmatrix}1&-2\\-3&5\end{pmatrix}$$

$$=\begin{pmatrix}-1&2\\3&-5\end{pmatrix}$$

（1）与式 $=\begin{pmatrix}8&3\\19&7\end{pmatrix}^{-1}$

$$=\frac{1}{8\cdot7-3\cdot19}\begin{pmatrix}7&-3\\-19&8\end{pmatrix}$$

$$=-\begin{pmatrix}7&-3\\-19&8\end{pmatrix}$$

$$=\begin{pmatrix}\mathbf{-7}&\mathbf{3}\\\mathbf{19}&\mathbf{-8}\end{pmatrix}$$

（2）与式 $=\begin{pmatrix}-1&2\\3&-5\end{pmatrix}\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$

$$=\begin{pmatrix}-3-4&1+2\\9+10&-3-5\end{pmatrix}$$

$$=\begin{pmatrix}\mathbf{-7}&\mathbf{3}\\\mathbf{19}&\mathbf{-8}\end{pmatrix}$$

（3）与式 $=\begin{pmatrix}3&-1\\-2&1\end{pmatrix}\begin{pmatrix}-1&2\\3&-5\end{pmatrix}$

$$=\begin{pmatrix}-3-3&6+5\\2+3&-4-5\end{pmatrix}$$

$$=\begin{pmatrix}\mathbf{-6}&\mathbf{11}\\\mathbf{5}&\mathbf{-9}\end{pmatrix}$$