新 基礎数学　改訂版，大日本図書

5 章 三角関数

2　三角関数

BASIC

■285(1)

$$630^\circ = 360^\circ + 270^\circ$$

■285(2)

$$-310^\circ = -360^\circ + 50^\circ$$

■285(3)

$$490^\circ = 360^\circ + 130^\circ$$

■285(4)

$$-1120^\circ = -1080^\circ - 40^\circ$$

■285(5)

$$2000^\circ = 1800^\circ + 200^\circ$$

■286(1)

第 3 象限

■286(2)

$400^\circ = 360^\circ + 40^\circ$　第 1 象限

■286(3)

$-740^\circ = -720^\circ - 20^\circ$　第 4 象限

■286(4)

$820^\circ = 720^\circ + 100^\circ$　第 2 象限

■286(5)

$-635^\circ = -720^\circ + 85^\circ$　第 1 象限

■287(1)

$$\sin 210^\circ = -\frac{1}{2}$$

■287(2)

$$\cos 570^\circ = \cos 210^\circ = -\frac{\sqrt{3}}{2}$$

■287(3)

$$\tan 390^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

■287(4)

$$\sin 630^\circ = \sin 270^\circ = -1$$

■287(5)

$$\cos(-225^\circ) = \cos 135^\circ = -\frac{1}{\sqrt{2}}$$

■287(6)

$$\tan(-480^\circ) = \tan(-120^\circ) = \sqrt{3}$$

■288 (1)

$$135^\circ = \frac{3}{4}\pi$$

■288 (2)

$$36^\circ = \frac{\pi}{5}$$

■288 (3)

$$-10^\circ = -\frac{1}{18}\pi$$

■288 (4)

$$240^\circ = \frac{4}{3}\pi$$

■288 (5)

$$-190^\circ = -\frac{19}{18}\pi$$

■289 (1)

$$\frac{\pi}{3} = 60^\circ$$

■289 (2)

$$\frac{5}{4}\pi = 225^\circ$$

■289 (3)

$$-\frac{2}{5}\pi = -72^\circ$$

■289 (4)

$$\frac{7}{3}\pi = 420^\circ$$

■289 (5)

$$-\frac{\pi}{9} = -20^\circ$$

■290 (1)

$$\sin\frac{5}{3}\pi = -\frac{\sqrt{3}}{2}$$

■290 (2)

$$\cos\frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$$

■290 (3)

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

■291 (1)

$$\begin{aligned} \text{弧の長さ：}\ 8\pi \times \frac{\frac{\pi}{6}}{2\pi} &= \frac{2}{3}\pi \\ \text{面積：}\ 16\pi \times \frac{\frac{\pi}{6}}{2\pi} &= \frac{4}{3}\pi \end{aligned}$$

■291 (2)

中心角を θ とすると

$$\begin{aligned} 6\pi \times \frac{\theta}{2\pi} = 2\pi &\implies \theta = \frac{2}{3}\pi \\ \text{面積：}\ 9\pi \times \frac{\frac{2}{3}\pi}{2\pi} &= 3\pi \end{aligned}$$

■292 (1)

$$\begin{aligned} (\text{左辺}) &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \\ &= \frac{1}{\sin\theta\cos\theta} = (\text{右辺}) \end{aligned}$$

■292 (2)

$$\begin{aligned} (\text{左辺}) &= \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \\ &= \frac{(1+\cos\theta)+(1-\cos\theta)}{1-\cos^2\theta} \\ &= \frac{2}{\sin^2\theta} = (\text{右辺}) \end{aligned}$$

■293 (1)

$180^\circ < \theta < 270^\circ$ なので $\cos\theta < 0$

$$\begin{aligned} \cos\theta &= -\sqrt{1-\left(-\frac{4}{5}\right)^2} = -\frac{3}{5} \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{4}{3} \end{aligned}$$

■293 (2)

$270^\circ < \theta < 360^\circ$ なので $\sin\theta < 0$

$$\begin{aligned} \sin\theta &= -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\frac{2\sqrt{2}}{3} \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} = -2\sqrt{2} \end{aligned}$$

■293 (3)

$$\begin{aligned} 1+\tan^2\theta &= \frac{1}{\cos^2\theta} \\ 1+9 &= \frac{1}{\cos^2\theta} \implies \cos^2\theta = \frac{1}{10} \end{aligned}$$

$180^\circ < \theta < 270^\circ$ なので $\cos\theta < 0$

$$\begin{aligned} \cos\theta &= -\frac{1}{\sqrt{10}} \\ \sin\theta &= \tan\theta\cdot\cos\theta = -\frac{3}{\sqrt{10}} \end{aligned}$$

■294 (1)

$$\begin{aligned} &\cos\theta\sin\left(\frac{\pi}{2}-\theta\right) + \sin\theta\cos\left(\frac{\pi}{2}-\theta\right) \\ &= \cos\theta\cdot\cos\theta + \sin\theta\cdot\sin\theta \\ &= \cos^2\theta + \sin^2\theta \\ &= 1 \end{aligned}$$

■294 (2)

$$\begin{aligned}&\sin\left(\frac{\pi}{2}-\theta\right)-\sin(\pi+\theta)+\\&\cos\left(\frac{\pi}{2}+\theta\right)+\cos(\pi-\theta)\\&=\cos\theta-(-\sin\theta)+(-\sin\theta)+(-\cos\theta)\\&=\cos\theta+\sin\theta-\sin\theta-\cos\theta\\&=0\end{aligned}$$

■294 (3)

$$\begin{aligned}&\tan(\pi+\theta)\sin\left(\frac{\pi}{2}+\theta\right)+\\&\cos(\pi-\theta)\tan(\pi-\theta)\\&=\tan\theta\cdot\cos\theta+(-\cos\theta)\cdot(-\tan\theta)\\&=\sin\theta+\sin\theta\\&=2\sin\theta\end{aligned}$$

■295 (1)

$$\begin{aligned}y&=\sin\left(x-\frac{\pi}{2}\right)\\&=-\cos x\end{aligned}$$

周期 :2π

■295 (2)

$$y=\cos\left(x+\frac{\pi}{4}\right)$$

周期 :2π

■296 (1)

$$y=2\cos x$$

周期 :2π

■296 (2)

$$y=-\frac{1}{2}\sin x$$

周期 :2π

■297 (1)

$$y=\cos 2x$$

周期 :π

■297 (2)

$$y=\sin\frac{x}{3}$$

周期 :6π

■298 (1)

$$\sin x=\frac{\sqrt{2}}{2}$$

$0\leqq x<2\pi$ より

$$x=\frac{\pi}{4},\frac{3}{4}\pi$$

■298 (2)

$$\cos x=\frac{1}{2}$$

$0\leqq x<2\pi$ より

$$x=\frac{\pi}{3},\frac{5}{3}\pi$$

■298 (3)

$$\sin x\geqq\frac{\sqrt{3}}{2}$$

$0\leqq x<2\pi$ より

$$\frac{\pi}{3}\leqq x\leqq\frac{2}{3}\pi$$

■298 (4)

$$\cos x<-\frac{1}{\sqrt{2}}$$

$0\leqq x<2\pi$ より

$$\frac{3}{4}\pi<x<\frac{5}{4}\pi$$

■299 (1)

$$\tan x=0$$

$0\leqq x<2\pi$ より

$$x=0,\pi$$

■299 (2)

$$\tan x=-1$$

$0\leqq x<2\pi$ より

$$x=\frac{3}{4}\pi,\frac{7}{4}\pi$$

CHECK

■300 (1)

$$40^\circ = \frac{40}{180}\pi = \frac{2}{9}\pi$$

■300 (2)

$$50^\circ = \frac{50}{180}\pi = \frac{5}{18}\pi$$

■300 (3)

$$-18^\circ = -\frac{18}{180}\pi = -\frac{1}{10}\pi$$

■300 (4)

$$-210^\circ = -\frac{210}{180}\pi = -\frac{7}{6}\pi$$

■301 (1)

$$-\frac{\pi}{4} = -45^\circ$$

■301 (2)

$$\frac{2}{3}\pi = 120^\circ$$

■301 (3)

$$-\frac{11}{6}\pi = -330^\circ$$

■301 (4)

$$\frac{7}{5}\pi = \frac{7}{5} \times 180^\circ = 252^\circ$$

■302 (1)

$$\sin(-90^\circ) = -1$$

■302 (2)

$$\cos 225^\circ = -\frac{1}{\sqrt{2}}$$

■302 (3)

$$\tan(-780^\circ) = \tan(-60^\circ) = -\sqrt{3}$$

■302 (4)

$$\sin\left(-\frac{17}{3}\pi\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

■302 (5)

$$\cos\frac{17}{6}\pi = \cos\frac{5}{6}\pi = -\frac{\sqrt{3}}{2}$$

■302 (6)

$$\tan\left(\frac{9}{4}\pi\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

■303

$$\text{弧の長さ：}\ 10\pi \times \frac{\frac{\pi}{4}}{2\pi} = \frac{5}{4}\pi$$
$$\text{面積：}\ 25\pi \times \frac{\frac{\pi}{4}}{2\pi} = \frac{25}{8}\pi$$

■304

$$\begin{aligned}(\text{左辺}) &= \frac{\sin\theta\{(1+\cos\theta)-(1-\cos\theta)\}}{(1-\cos\theta)(1+\cos\theta)} \\ &= \frac{2\sin\theta\cos\theta}{1-\cos^2\theta} \\ &= \frac{2\sin\theta\cos\theta}{\sin^2\theta} \\ &= \frac{2}{\tan\theta} = (\text{右辺})\end{aligned}$$

■305

$180^\circ < \theta < 270^\circ$ なので $\cos\theta < 0$

$$\begin{aligned}\cos\theta &= -\sqrt{1-\left(-\frac{1}{4}\right)^2} \\ &= -\frac{\sqrt{15}}{4}\end{aligned}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{15}}$$

■306 (1)

$$y = \cos\left(x - \frac{\pi}{6}\right)$$

周期：2π

■306 (2)

$$y = 2\sin 3x$$

周期：$\frac{2}{3}\pi$

■307 (1)

$$\sin x = -\frac{\sqrt{2}}{2}$$

$0 \leqq x < 2\pi$ より

$$x = \frac{5}{4}\pi,\ \frac{7}{4}\pi$$

■307 (2)

$$\tan x = -\sqrt{3}$$

$0 \leqq x < 2\pi$ より

$$x = \frac{2}{3}\pi,\ \frac{5}{3}\pi$$

■307 (3)

$$\begin{aligned} 2\sin x - 1 &< 0 \\ \sin x &< \frac{1}{2} \end{aligned}$$

$0 \leqq x < 2\pi$ より

$$0 \leqq x < \frac{\pi}{6},\ \frac{5}{6}\pi < x < 2\pi$$

■307 (4)

$$\cos x \leqq \frac{\sqrt{2}}{2}$$

$0 \leqq x < 2\pi$ より

$$\frac{\pi}{4} \leqq x \leqq \frac{7}{4}\pi$$

■308

$$y = \sin x$$

(1) a の値を求める：

$$a = \sin\frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$$

(2) b の値を求める（$0 < b < \frac{5}{4}\pi$）：

$$\frac{1}{2} = \sin b \implies b = \frac{5}{6}\pi$$

(3) c の値を求める（$-\pi < c < 0$）：

$$-1 = \sin c \implies c = -\frac{\pi}{2}$$

STEP UP

■309

$$\begin{aligned}
&OA : OB = r_1 : r_2 \\
&r_2 \cdot OA = r_1 \cdot (OA + l) \\
&OA \cdot (r_2 - r_1) = r_1 l \\
&OA = \frac{r_1}{r_2 - r_1} l, \quad OB = OA + l = \frac{r_2}{r_2 - r_1} l
\end{aligned}$$

側面の展開図のおうぎ形の中心角を θ とすると

$$\begin{aligned}
&2\pi r_1 = 2OA \cdot \pi \cdot \frac{\theta}{2\pi} \\
&\theta = \frac{r_1}{OA} \cdot 2\pi = \frac{2(r_2 - r_1)}{l}\pi \\
&S = \pi \cdot OB^2 \cdot \frac{\theta}{2\pi} - \pi \cdot OA^2 \cdot \frac{\theta}{2\pi} \\
&= \frac{\theta}{2}\left(\frac{r_2 l}{r_2 - r_1}\right)^2 - \frac{\theta}{2}\left(\frac{r_1 l}{r_2 - r_1}\right)^2 \\
&= \frac{\pi(r_2 - r_1)}{l} \cdot \left(\frac{l}{r_2 - r_1}\right)^2 (r_2^2 - r_1^2) \\
&= \frac{\pi l}{r_2 - r_1} \cdot (r_2 + r_1)(r_2 - r_1) \\
&= \pi l (r_1 + r_2)
\end{aligned}$$

■310

半径を r とすると弧の長さは $12 - 2r \quad (0 < r < 6)$

中心角を θ とおくと

$$12 - 2r = r\theta \implies \theta = \frac{12 - 2r}{r}$$

面積 S は

$$\begin{aligned}
&S = \frac{1}{2} r^2 \theta \\
&= \frac{1}{2} r^2 \left(\frac{12 - 2r}{r}\right) \\
&= 6r - r^2 \\
&= -(r - 3)^2 + 9
\end{aligned}$$

$r = 3$ で最大値 9 をとる

■311

解と係数の関係より

$$\begin{cases} \sin\theta + \cos\theta = \frac{2}{3} \\ \sin\theta\cos\theta = \frac{k}{3} \end{cases}$$

$(\sin\theta + \cos\theta)^2 = \frac{4}{9}$ より

$$\begin{aligned}
&1 + 2\sin\theta\cos\theta = \frac{4}{9} \\
&2 \cdot \frac{k}{3} = -\frac{5}{9} \\
&k = -\frac{5}{6}
\end{aligned}$$

■312 (1)

$$\begin{aligned}
y &= \sin\left(2x - \frac{\pi}{2}\right) \\
&= -\cos 2x
\end{aligned}$$

周期：π

■312 (2)

$$y = \frac{1}{2}\cos\left\{3\left(x - \frac{\pi}{12}\right)\right\}$$

$y = \frac{1}{2}\cos 3x$ のグラフを x 軸方向に $\frac{\pi}{12}$ だけ平行移動したもの

周期：$\frac{2}{3}\pi$

■312 (3)

$$y = -\tan\left\{2\left(x - \frac{\pi}{4}\right)\right\}$$

$y = -\tan 2x$ のグラフを x 軸方向に $\frac{\pi}{4}$ だけ平行移動したもの

周期：$\frac{\pi}{2}$

■313 (1)

$$\begin{aligned}
2\cos\left(x + \frac{\pi}{3}\right) &= \sqrt{3} \\
\cos\left(x + \frac{\pi}{3}\right) &= \frac{\sqrt{3}}{2}
\end{aligned}$$

$0 \leqq x < 2\pi$ より $\frac{\pi}{3} \leqq x + \frac{\pi}{3} < \frac{7}{3}\pi$ なので

$$\begin{aligned}
x + \frac{\pi}{3} &= \frac{11}{6}\pi,\ \frac{13}{6}\pi \\
x &= \frac{3}{2}\pi,\ \frac{11}{6}\pi
\end{aligned}$$

■313 (2)

$$\sin 2x = \frac{1}{2}$$

$0 \leqq x < 2\pi$ より $0 \leqq 2x < 4\pi$ なので

$$\begin{aligned} 2x &= \frac{\pi}{6},\ \frac{5}{6}\pi,\ \frac{13}{6}\pi,\ \frac{17}{6}\pi \\ x &= \frac{\pi}{12},\ \frac{5}{12}\pi,\ \frac{13}{12}\pi,\ \frac{17}{12}\pi \end{aligned}$$

■313 (3)

$$\begin{aligned} 2\sin\left(2x - \frac{\pi}{6}\right) &= 1 \\ \sin\left(2x - \frac{\pi}{6}\right) &= \frac{1}{2} \end{aligned}$$

$0 \leqq x < 2\pi$ より $-\frac{\pi}{6} \leqq 2x - \frac{\pi}{6} < \frac{23}{6}\pi$ なので

$$\begin{aligned} 2x - \frac{\pi}{6} &= \frac{\pi}{6},\ \frac{5}{6}\pi,\ \frac{13}{6}\pi,\ \frac{17}{6}\pi \\ 2x &= \frac{\pi}{3},\ \pi,\ \frac{7}{3}\pi,\ 3\pi \\ x &= \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{7}{6}\pi,\ \frac{3}{2}\pi \end{aligned}$$

■314

$$\tan x \leqq \sqrt{3}$$

$0 \leqq x < 2\pi$ より

$$0 \leqq x < \frac{\pi}{3},\ \frac{\pi}{2} < x \leqq \frac{4}{3}\pi,\ \frac{3}{2}\pi < x < 2\pi$$

■315 (1)

$$\begin{aligned} 2\cos^2 x + \sin x - 1 &= 0 \\ 2(1 - \sin^2 x) + \sin x - 1 &= 0 \\ 2\sin^2 x - \sin x - 1 &= 0 \\ (2\sin x + 1)(\sin x - 1) &= 0 \\ \sin x &= -\frac{1}{2},\ 1 \end{aligned}$$

$0 \leqq x < 2\pi$ より

$$x = \frac{\pi}{2},\ \frac{7}{6}\pi,\ \frac{11}{6}\pi$$

■315 (2)

$$\begin{aligned} 2\sin^2 x + 5\cos x - 4 &< 0 \\ 2(1 - \cos^2 x) + 5\cos x - 4 &< 0 \\ 2\cos^2 x - 5\cos x + 2 &> 0 \\ (2\cos x - 1)(\cos x - 2) &> 0 \\ \cos x < \frac{1}{2},&\ 2 < \cos x \end{aligned}$$

$0 \leqq x < 2\pi$ かつ $-1 \leqq \cos x \leqq 1$ より

$$\frac{\pi}{3} < x < \frac{5}{3}\pi$$

■316 (1)

$$y = \sin^2 x - \sin x - 1$$

$t = \sin x$ とおくと、$0 \leqq x < 2\pi$ より $-1 \leqq t \leqq 1$

$$\begin{aligned} y &= t^2 - t - 1 \\ &= \left(t - \frac{1}{2}\right)^2 - \frac{5}{4} \end{aligned}$$

■316 (2)

$-1 \leqq t \leqq 1$ において

$t = -1$ すなわち $x = \frac{3}{2}\pi$ で最大値 1

$t = \frac{1}{2}$ すなわち $x = \frac{\pi}{6},\ \frac{5}{6}\pi$ で最小値 $-\frac{5}{4}$

■317 (1)

$$\begin{cases} 2\sin x - 1 > 0 \\ 2\cos x - \sqrt{2} \leqq 0 \end{cases}$$

$$\begin{cases} \sin x > \frac{1}{2} \\ \cos x \leqq \frac{1}{\sqrt{2}} \end{cases}$$

$$\begin{cases} \frac{\pi}{6} < x < \frac{5}{6}\pi \\ \frac{\pi}{4} \leqq x \leqq \frac{7}{4}\pi \end{cases}$$

$$\iff \frac{\pi}{4} < x < \frac{5}{6}\pi$$

■317 (2)

$$\begin{cases} \tan x + 1 < 0 \\ 2\cos x < 1 \end{cases}$$

$$\begin{cases} \tan x < -1 \\ \cos x < \frac{1}{2} \end{cases}$$

$$\begin{cases} \frac{\pi}{2} < x < \frac{3}{4}\pi,\ \frac{3}{2}\pi < x < \frac{7}{4}\pi \\ \frac{\pi}{3} < x < \frac{5}{3}\pi \end{cases}$$

$$\iff \frac{\pi}{2} < x < \frac{3}{4}\pi,\ \frac{3}{2}\pi < x < \frac{5}{3}\pi$$