QUANTITATIVE JUSTIFICATION FOR PROJECT AUTHORIZATION TO PROCEED

Norikazu Hara
National Space Development Agency of Japan
Tokyo, Japan
Hara-Norikazu@gakushikai.jp

ABSTRUCT

The value of mission is the converted monetary value from all of the benefit from the successful mission. Cost includes all of the necessary expense for development and operation. Risk is the expectation of loss, which includes not only direct but also indirect loss incurred by the mission failure. The concept of utility could be considered in both the mission value and the loss. The probability of mission failure, which is one of two components of risk, is the degree of belief in the proposition that the mission will end in failure. The concept of probability necessary for risk evaluation is the degree of belief. By the concept and by recognizing that equal distribution is no more than the expression for no information, we can derive the Laplace's Rule of Succession naturally. Recognizing risk is a kind of quantity, which has the unit of the value, we can derive a simple inequality that mission value should be larger than the summation of cost and risk. This inequality is a necessary condition for the justification for the project ATP. To show the use of this inequality, a virtual project will be assessed using some data created by the author's imagination.

PREFACE

Generally speaking when we develop some items for a new mission it is natural to conduct feasibility study thoroughly before we start work. We need to confirm the feasibility of development. However, whatever extent we conduct research and study, it is inevitable to remain the risk that we may fail to develop it. It is inevitable for all of the activities to be accompanied with risk. In addition to the condition that necessary expense (cost) is lower than the budget, we have to consider the justification for giving the Authorization To Proceed (ATP) under the risk situation balancing the benefit from the successful mission. In concept this consideration has been called as "Risk and Benefit" so far. However, as is shown in the Superconducting Super Collider (SSC) project [1], it has not been assessed quantitatively except cost estimation.

Looking at another example, a tunnel digging construction, even after its completion there remains severe criticism that too much over cost and too many sacrifices were incurred for the useless tunnel, because the traffic circumstances has been greatly changed during the long term of construction. It is unhappy for the promoters of the project but also for the taxpayers to be regretted for not stopping the works even in the midst of the project.

The author suggested the justification condition for the project authorization to proceed could be obtained by recognizing explicitly that risk had the unit of value [2]. This paper presents the new way of inducing the Laplace's Rule of Succession to show the concept of degree of belief is appropriate for the definition of the probability and also presents how to use of the justification condition for the ATP with an example of virtual project.

DEFINITION OF RISK

It is essential for the risk to compare its high or low. For this risk must be defined as a measurable quantity not as an ambiguous one. In addition to prepare the rational countermeasure risk is desirable to be an additive quantity. Accordingly the author asserted risk must be defined as the expectation of loss and mentioned the importance of recognition that risk has the unit of value [2]. If loss is measured with unit of value, the author emphasized that the monetary unit can be used as it is and that risk has also measured with monetary unit. The monetary unit as the unit of value is additive. The author also stressed that the meaning of the probability in the assessment of risk must be degree of belief, which is the original meaning of probability. Mathematical theory of probability is a theoretical system, which starts from axiom satisfying additive rule. Risk, which is a multiplication product of value and probability, has also unit of value and is also additive. Quantitative Risk Assessment (QRA) is executable under these backgrounds.

VALUE OF MISSION

Motivation to develop something is born by our recognition to the value of mission attained by developing and operating it. The value of mission is the benefits from executing the mission. It is appropriate to use monetary unit for expressing the value if it has large value or not. These values have not been necessary, if more exactly speaking never assessed by, the quantitative expressions. However when the decision was made to proceed, it may be certain that they recognized the value would be larger than at least the cost for development.

Let us consider a simple case when we purchase a commodity, we will not buy it if we think the price is too high. However, there is another case that we buy it even if its price is much higher than the regular price we think appropriate. Those people, who are compelled to need it, recognize it valuable much higher than the price. This fact tells that real value is different among people who are in different situation. The value of mission is also different among people. Therefore, we need to include the concept of utility [3] for the evaluation of mission value. As the benefit of mission is different according to the imagination to the effects of the success of mission, the value of mission cannot have a meaning more exact value than a rough estimation.

It is not unusual when the imagination become unreal, but it will be certain that the company producing always lower value products than the cost will collapse. It must not be allowed to proceed a project knowing its mission value is lower than cost from the beginning. In addition as development is always accompanied with a risk, it is necessary to add the amount of risk to the cost when we make a decision to proceed the development. The value of mission never been estimated exactly. Instead, it must be expressed how much the person concerned took the value of it considering its utility.

[the value of mission] = [the value of mission recognized by most of people (average)] + [the attached value to the person concerned (considering utility)] …(1)

DEVELOPMENT COST ESTIMATION

In this paper the meaning of cost includes the all of necessary expense, which means the total cost. Any items are rarely developed without a care of its cost. In the most of case, as seen in Japanese space development project for example, the development cost is sure to be estimated in advance. Apparently the development will certainly fail if its cost will not be able to be covered. The total cost is the summing up all of the expense assuming the development goes without trouble. The budget is made up from the total cost estimation. Although budget is estimated in detail numerical value in many cases, naturally cost does not have the meaning of more than rough estimation either.

Although it is desirable to prepare the reserve fund for unforeseeable troubles in the budget, practically it is difficult for various reasons. Therefore it is often the case that additional cost is incurred after starting the development.

PROBABILITY OF DEVELOPMENT FAILURE

After the development, it is not necessary certain that the development item will work as expected. There is always possibility of failure in the newly development item. The success probability is what is expressed in the numerical value for the degree of certainty on the success of the development. Conversely the failure probability is what is expressed numerically on the degree of certainty on the failure.

[the success probability] + [the failure probability] = 1 …(2)

Where we need to remind that the mission concerned is only one time event and that those probabilities are the degree of belief and subjective probabilities. Therefore the failure probability can be different among persons. For the development case this probability should be the degree of belief of project manager and the belief should be accountable to others. The degree of belief should not be too high or too low without any reason. It should be as the way of assigning the same number if the same information would be brought. In other word, the number must be what agreed by others if it is explained. If your degree of belief is different from mine, you and I should show the basis to each other for the number and try to be able to assign same number as possible as we can.

It seemed to me that the assignment probability is much difficult for us than the estimation of cost. This is because we are not accustomed to express the degree of belief with numerical value. Especially we are apt to see the very small probability larger than as is.

DEGREE OF BELIEF AND LAPLACE’S RULE OF SUCCESSION

As the equation for assignment probability after getting information, we have Laplace’s Rule of Succession in the simplest case. After getting the information that total number of sample is n and the number of success is r, we can assign the number for degree of belief P for next success with the following equation.

$$P = \frac{r+1}{n+2} \qquad \dots (3)$$

After Laplace firstly induced this equation, there was the critique to the equal distribution assumption for a priori distribution [4]. However, the same equation can be induced by recognizing that the equal distribution is another expression for no information situation and by using the Bayes's Theory [5].

PROBERVILITY AS DEGREE OF BELIEF

We adopt the definition of probability as degree of belief. It was established by Savage that degree of belief could be defined as a subjective probability. The probability as degree of belief is no more than the numerical expression for the state of human mind. The probability (p) means "degree of belief for the truth on the proposition." That is, the expression using numerical value from 0 to 1 for the degree of certainty for the truth on the proposition. The value of p can be given any value between 0 and 1. However, the following three values are special cases.

$p \mapsto 1$ ：extremely strong belief of the truth on the proposition. （a symbol, $p \mapsto 1$ means that numerical value 1 is given to p）

$p \mapsto 0$ ：extremely strong belief of the false on the proposition.

$p \mapsto 0.5$ ：entirely no confident about the truth on the proposition, so called, fifty-fifty.

Let us consider the experiment that we pick a stone from urn, which contains white and black stones. The proposition is "the stone picked is white". It is our problem what is the degree of belief, that is, probability (p), for the truth on the proposition.

If we saw the fact that only white stones are packed into the urn, then $p \mapsto 1$. If only black stones, $p \mapsto 0$. If we knew 30 black and 70 white stones were packed into the urn, then, $p \mapsto 0.7$. If we knew same number of white and black stones are packed, then, $p \mapsto 0.5$. These numerical values are the probability by Laplace's definition and with strong belief on the value of p.

To the next, suppose the case we do not know how much white and black stones were packed into the urn at all. Even such a case, the stone picked from urn must be white or black. In this case also the probability (p) of truth on the proposition is $p \mapsto 0.5$.

When we knew the same number of white and black stones are packed, it was also same $p \mapsto 0.5$. The difference between these two cases is the contents to p, that is, density of belief on p. The difference is the shape of density of belief on p, π (p).

Based on the fact that both case we give $p \mapsto 0.5$, it is concluded as appropriate that "the expectation of the density of the probability is equal to its probability". This should be taken as a principle accompanied the definition of probability as degree of belief. In other words, we adopt the expectation for converting equation necessary for representing a value for the probability expressed with distribution form. Now, we will have no confusion for writing [=] for ［$\mapsto$］ instead of writing $p \mapsto E(p)$.

$$p = E(p) = \int_0^1 p\pi(p)dp \qquad \dots (4)$$

The former is the case we can have strong belief on p because we knew the stones packed. We can express this density of belief distribution with Dirac's delta function (Fig.1).

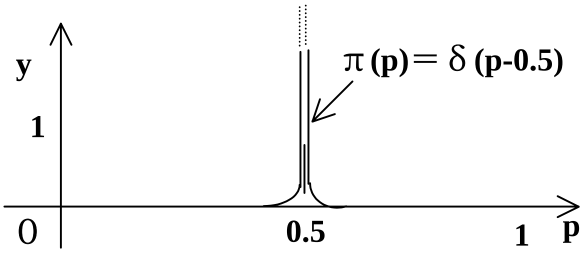

Fig. 1　Strong Density of Belief on p

When we knew same number of white and black stones is packed,

$$\pi(p) = \delta(p - 0.5) \qquad \ldots (5)$$

Conversely, the latter is the case we did not know how stones were packed. It is the weakest degree of belief and we can express the function of density equal distribution from 0 to 1 (Fig. 2).

$$\pi(p) = 1 \qquad \ldots (6)$$

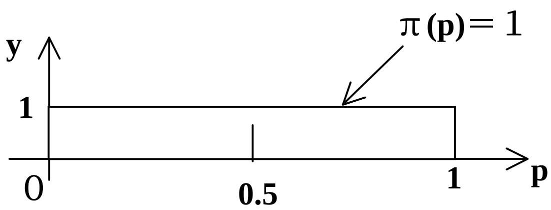

Fig. 2 Weak Density of Belief on p

Even in the latter case, gradually the belief will get stronger by picking up stones one by one. That is, by seeing data the shape of density of belief will be deformed.

Density of belief π (p) has the following natures.

$$\left.\begin{array}{ll} \pi(p) = 0, & p < 0, \quad p > 1 \\ \pi(p) \geq 0, & 0 \leq p \leq 1 \end{array}\right\} \qquad \ldots (7)$$

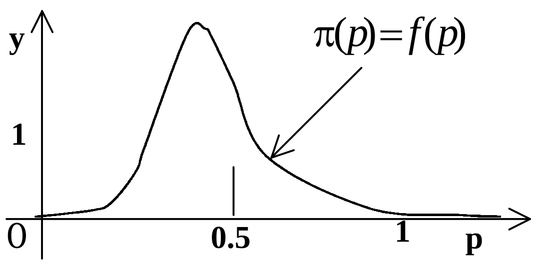

Fig. 3 General Density of Belief on p

In addition, we can add the equation as the condition for normalization.

$$\int_{-\infty}^{\infty} \pi(p)dp = \int_{0}^{1} \pi(p)dp = 1 \qquad \ldots (8)$$

Fig. 3 shows the general density of belief on p.

<u>PROBABILITY BY LAPLACE AND D'ALENBERT</u> Let us consider the experiment of picking out two stones independently from the previous urn. Letting p for the probability of white stone from urn, the probability of both white is p^2 . If the density of belief on p is expressed as π (p), then the equation (4) turns to the equation (9).

$$E(W,W) = {}_2C_2 \int_0^1 p^2 \pi(p) dp \qquad \ldots (9)$$

Similarly, in case of one is white and the other is black and in case of both black the probability is expressed by the equations (10) and (11) respectively.

$$E(W,B) = {}_2C_1 \int_0^1 p(1-P) \pi(p) dp \qquad \ldots (10)$$

$$E(B,B) = {}_2C_0 \int_0^1 (1-P)^2 \pi(p) dp \qquad \ldots (11)$$

If we enter equation (5) for π (p) in (9), (10), and (11) and if calculate them, we get values of 1/4、1/2, 1/4 respectively. This corresponds to the Binomial distribution B(2,0.5).

On the other hand, entering equation (6) to π (p), we get values of 1/3、1/3、1/3 respectively. This result shows equal distribution of 1/3 to the possible three states.

Tossing two coins simultaneously, the probabilities of both heads, one is head and the other is tail, and both tail, were determined as 1/3,1/3, and 1/3 by D'Alenbert. It is reported that Laplace who was a D'Alenbert's pupil corrected them as 1/4, 1/2, and 1/4 respectively. Each coin has head and tail. Therefore this is the case of strong belief on p=0.5. In the real experiment, the statistics will show the nearly Laplace's probability.

Besides, in the case of picking out n coins independently from urn we can conduct same calculation. Final result will be Binomial distribution B(n,0.5) or equal distribution depending on π (p).

DEGREE OF BELIEF AFTER GETTING DATA If degree of belief is strong enough as shown with Dirac's delta function, the probability is unchangeable by seeing finite number of data. However, if a priori belief is weak, the degree of belief, that is probability, will change by seeing data. This change can be calculated using Bayes's Theorem.

Bayes's Theorem asserts the following [6]. "A posteriori density after observing data is proportional to the product of likelihood of data and a priori density."

The probability p after observing data is the expectation of a posteriori density on p.

Inspection by attribute is a test which provides data of only success or fail. If we take success for white stone, it corresponds to the experiment of picking out stones from urn of the case we did not see the packing of stones. All of stones may be white or fairy number of black stones might be mixed.

Firstly, for the initial state it is appropriate by the previous consideration to adopt equation (6) for a priori density of belief.

Data X (x1, x2, …, xn) means a series of success, and fail. As these are independent, the order is no relation to the probability but only the number of test samples, n, and the number of success r affects a posteriori density.

After observing data X, a posteriori density, π (p|X) ,

$$\pi\left(p\middle|X\right)\propto L\left(X\middle|p\right)\pi\left(p\right) \qquad \ldots(12)$$

Where, L(X|p) is the likelihood of data X on p.

The likelihood of data, r success among n samples, is $p^r(1-p)^{n-r}$. Then,

$$\pi(p\mid X)\propto p^r(1-p)^{n-r}\times 1 \qquad \ldots(13)$$

From the condition of equation (8), we can determine the constant utilizing Beta integral formula.

$$\pi(p\mid X)=\frac{\Gamma(n+2)}{\Gamma(r+1)\Gamma(n-r+1)}p^r(1-p)^{n-r} \qquad \ldots(14)$$

Equation (14) is no more than a β -distribution. Therefore, the degree of belief after observing data, that is, probability p is given by the expectation of equation (4).

$$p=E(\pi(p\mid X))=\frac{r+1}{n+2} \qquad \ldots(15)$$

Where, n is number of test sample, r is number of success among n. Equation (15) is the probability we seek after observing data. This is known as Laplace's Rule of Succession, previously mentioned as equation (3).

LOSS ACCOMPANIED BY FAILURE OF DEVELOPMENT

When we make a decision on the proceeding the development, we have to estimate the loss incurred from the failure of development. Firstly what we lose by failure is the benefit from success of mission, that is, the value of mission mentioned previously. In addition, we will have the parasite loss such as bad reputation caused by the failure. This depends on the situation of the organization. It may be called "minus utility". There may be the case that small project, which gives a little benefit but small cost, produces equivalently large loss by losing credit of company if it fails. The loss we incurred if it fails is the summation of both value and loss.

[the loss incurred by failure] = [the value of mission] + [parasite loss accompanied by failure] … (16)

NESSARY CONDITION FOR ATP

In the case of buying and selling of daily commodity the buyer put the price to the item. If the buyer who needs the item think the value of the item for him is higher than the price, and if he has enough money to purchase it, he will buy it. As the price of the item is higher than the value of it for the seller, then he agrees to sell it. Seller has the right of not selling it while buyer has the right of not buying it. The sales contract is achieved only when both sides think it profitable to do so.

In such ordinary commodity contract there is no risk. Therefore the condition for justifying the purchase for buyer is the value of the item for the buyer is higher than the price.

[the value of the item for the buyer] > [the price of the item] … (17)

The same condition exists for seller.
[the value of the item for the seller] < [the price of the item] … (18)

For the case of development the condition that the value of mission is higher than the cost is not enough for the justification to give the ATP. It is necessary to add the development risk to development cost.

[the value of mission] > [development cost] + [development risk] … (19)

Even this equation is satisfied it is not decided if we can proceed to the development in real. This is because the decision depends on the resources we have. The inequality (19) is just necessary condition for the justification for ATP.

NESSARY CONDITION FOR PROJECT CONTINUATION

After beginning to fund for the development if the development is on schedule as the plan for the project, the condition of justification for ATP will be maintained. However, it is often the case that the additional cost is requested or development risk is enlarged when the development does not go as planned. Even the value of mission may be changed during the extremely prolonged development. In this case the study for the continuation of the development would be required. The following equation will be the justification for the continuation of the development.

[the value of mission restudied] > [necessary expense from now] + [residual risk restudied] … (20)

In the most of cases the value of mission is unchanged. However for the mission where the time is critical the value of mission could be reduced extremely. The development risk would become large or small by restudy of the project. In the most of projects even if additional cost is required, as it must be smaller than initial cost might be spent, this equation is satisfied with ease. Therefore, it might be rightly criticized as the slovenly planned if the project were stopped in the middle of development.

If we find that the inequality (20) is not satisfied by review in the middle of development, we should stop developing. By stopping development we could save the development cost scheduled in the future even it was small amount.

EXAMPLE OF ATP JUSTIFICATION STUDY

As it is decided by getting concurrence of many people when development starts, it may not be necessary to show quantitatively that mission value is larger than the summation of cost and risk. However, when the development is prolonged much longer than as scheduled initially, it is recommended to make the justification for continuation of the development clear using inequality (20) in the middle of the development.

A virtual project has been prolonged greatly from initial schedule. As the example of the study, the study on the continuation of this project is shown in the table-1 followed. But you have to take these values in the table are virtual by the author.

This tables are better to be prepared by the person concerned (for example, project manager), the amount of risk as well as mission value are open to be criticized in public.

Table-1 Table for ATP Justification Study（Example）

Name of Project	ABCD Project			
Time of Evaluation	Transition to development 1995.4	At this point 2002.4	Symbol	Remark
Value of Mission (10^8yen)	10,000	8,000	A	1), 2), 3)
Cost (10^8yen)	6,000	3,000	B	4), 5)
Additional Loss when failure (10^8yen)	1,000	1,000	C	6)
Risk (10^8yen)	1,100	810	D	＝E x (A +C)
Probability of Failure (-)	0.1	0.09	E	7)
Condition of ATP	Yes	Yes	F	If A > B + D

1) Including utility for the development organization.
2) Shorten the mission term.
3) Initial expectation has been shrunk.
4) Total expense after the evaluation.
5) Including the expense of 7-year operation.
6) Converted estimation for unpopularity to the development organization when failed.
7) Good development test result.

CLOSING REMARKS

There may be such a case as additional cost is added every year after prolonged development. Finally development ended, it would be evaluated by the following inequality whether the project was truly succeeded or not.

[value of mission succeeded（after evaluation）]
> [total development cost] …(21)

If development schedule is very much prolonged, total development cost may be necessary to convert to the current value as simple addition may be inappropriate. However, to confirm the satisfaction of this inequality it is useless to seek the precision of these values in table. It is enough to express with one or two effective digit numbers.

REFERRENCE

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