

1 章 微分法

問 1

$$\begin{aligned}(1) \quad y' &= 5(x^2 - 1)^4 \cdot (x^2 - 1)' \\ &= 5(x^2 - 1)^4 \cdot 2x \\ &= 10x(x^2 - 1)^4\end{aligned}$$

$$\begin{aligned}(2) \quad y' &= \cos(x^2 + 1) \cdot (x^2 + 1)' \\ &= \cos(x^2 + 1) \cdot 2x \\ &= 2x \cos(x^2 + 1)\end{aligned}$$

$$\begin{aligned}(3) \quad y' &= x^{x^2} \cdot (x^2)' \\ &= e^{x^2} \cdot 2x \\ &= 2xe^{x^2}\end{aligned}$$

$$\begin{aligned}(4) \quad y &= (e^x + 1)^{\frac{1}{2}} \\ y' &= \frac{1}{2}(e^x + 1)^{-\frac{1}{2}} \cdot (e^x + 1)' \\ &= \frac{1}{2(e^x + 1)^{\frac{1}{2}}} \cdot e^x \\ &= \frac{e^x}{2\sqrt{e^x + 1}}\end{aligned}$$

問 2

$$\begin{aligned}(1) \quad y' &= 2 \sin x \cdot (\sin x)' \\ &= 2 \sin x \cdot \cos x \\ &= 2 \sin x \cos x \\ &= \sin 2x\end{aligned}$$

$$\begin{aligned}(2) \quad y' &= 3 \tan^2 x \cdot (\tan x)' \\ &= 3 \tan^2 x \cdot \frac{1}{\cos^2 x} \\ &= \frac{3 \tan^2 x}{\cos^2 x}\end{aligned}$$

問 3

$$\begin{aligned}(1) \quad y' &= 3 \tan^2 2x \cdot (\tan 2x)' \\ &= 3 \tan^2 2x \cdot \frac{1}{\cos^2 2x} \cdot (2x)' \\ &= \frac{3 \tan^2 2x}{\cos^2 2x} \cdot 2 \\ &= \frac{6 \tan^2 2x}{\cos^2 2x}\end{aligned}$$

$$\begin{aligned}(2) \quad y' &= -\sin \sqrt{x^2 + 1} \cdot (\sqrt{x^2 + 1})' \\ &= -\sin \sqrt{x^2 + 1} \cdot \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot (x^2 + 1)' \\ &= -\sin \sqrt{x^2 + 1} \cdot \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x \\ &= -\frac{x \sin \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}}\end{aligned}$$

$$\begin{aligned}(3) \quad y' &= \{(2x + 1)^3\}'(x^2 - x + 1)^2 \\ &\quad + (2x + 1)^3 \{(x^2 - x + 1)^2\}' \\ &= 3(2x + 1)^2(2x + 1)'(x^2 - x + 1)^2 \\ &\quad + (2x + 1)^3 \cdot 2(x^2 - x + 1)(x^2 - x + 1)' \\ &= 3(2x + 1)^2 \cdot 2(x^2 - x + 1)^2 \\ &\quad + 2(2x + 1)^3(x^2 - x + 1)(2x - 1) \\ &= 6(2x + 1)^2(x^2 - x + 1)^2 \\ &\quad + 2(2x + 1)^3(x^2 - x + 1)(2x - 1) \\ &= 2(2x + 1)^2(x^2 - x + 1) \\ &\quad \times \{3(x^2 - x + 1) + (2x - 1)(2x + 1)\} \\ &= 2(2x + 1)^2(x^2 - x + 1) \\ &\quad \times (3x^2 - 3x + 3 + 4x^2 - 1) \\ &= 2(2x + 1)^2(x^2 - x + 1)(7x^2 - 3x + 2)\end{aligned}$$

問 4

$$\begin{aligned}(1) \quad y' &= x' \cdot \log x + x(\log x)' \\ &= 1 \cdot \log x + x \cdot \frac{1}{x} \\ &= \log x + 1\end{aligned}$$

$$\begin{aligned}(2) \quad y' &= \frac{(\log x)' \cdot x - \log x \cdot x'}{x^2} \\ &= \frac{\frac{1}{x} \cdot x - \log x \cdot 1}{x^2} \\ &= \frac{1 - \log x}{x^2}\end{aligned}$$

$$\begin{aligned}(3) \quad y' &= \frac{1}{3x + 1} \cdot (3x + 1)' \\ &= \frac{3}{3x + 1}\end{aligned}$$

$$\begin{aligned}(4) \quad y' &= \frac{1}{x^2 + 1} \cdot (x^2 + 1)' \\ &= \frac{2x}{x^2 + 1}\end{aligned}$$

$$\begin{aligned} (5) \quad y' &= \frac{1}{\cos x} \cdot (\cos x)' \\ &= \frac{-\sin x}{\cos x} \\ &= -\tan x \end{aligned}$$

$$\begin{aligned} (6) \quad y' &= \frac{1}{e^x + e^{-x}} \cdot (e^x + e^{-x})' \\ &= \frac{1}{e^x + e^{-x}} \cdot \{e^x + e^{-x} \cdot (-x)'\} \\ &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \end{aligned}$$

問 5

$$\begin{aligned} (1) \quad y &= \log(x+1)^4 - \log(x-1)^3 \\ &= 4\log(x+1) - 3\log(x-1) \end{aligned}$$

よって

$$\begin{aligned} y' &= 4 \cdot \frac{1}{x+1} \cdot (x+1)' - 3 \cdot \frac{1}{x-1} \cdot (x-1)' \\ &= \frac{4}{x+1} - \frac{3}{x-1} \\ &= \frac{4(x-1) - 3(x+1)}{(x+1)(x-1)} \\ &= \frac{x-7}{(x+1)(x-1)} \end{aligned}$$

$$(2) \quad y = \log(\sin x) - \log x$$

よって

$$\begin{aligned} y' &= \frac{1}{\sin x} \cdot (\sin x)' - \frac{1}{x} \\ &= \frac{\cos x}{\sin x} - \frac{1}{x} \\ &= \frac{x \cos x - \sin x}{x \sin x} \end{aligned}$$

$$(3) \quad y = \log(x^2 + 1)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log(x^2 + 1)$$

よって

$$\begin{aligned} y' &= \frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot (x^2 + 1)' \\ &= \frac{2x}{2(x^2 + 1)} \\ &= \frac{x}{x^2 + 1} \end{aligned}$$

$$(4) \quad y = \log x(x^2 + 4)^{\frac{1}{3}}$$

$$= \log x + \log(x^2 + 4)^{\frac{1}{3}}$$

$$= \log x + \frac{1}{3} \log(x^2 + 4)$$

よって

$$\begin{aligned} y' &= \frac{1}{x} + \frac{1}{3} \cdot \frac{1}{x^2 + 4} \cdot (x^2 + 4)' \\ &= \frac{1}{x} + \frac{2x}{3(x^2 + 4)} \\ &= \frac{3(x^2 + 4) + 2x \cdot x}{3x(x^2 + 4)} \\ &= \frac{5x^2 + 12}{3x(x^2 + 4)} \end{aligned}$$

問 6

$y = x^\alpha$ とおき, 両辺の自然対数をとると

$$\log y = \log x^\alpha$$

$$= \alpha \log x$$

両辺を x で微分すると

$$\frac{d}{dy}(\log y) \frac{dy}{dx} = \alpha \cdot \frac{1}{x}$$

$$\frac{1}{y} \cdot y' = \frac{\alpha}{x}$$

よって

$$\begin{aligned} y' &= y \cdot \frac{\alpha}{x} \\ &= x^\alpha \cdot \frac{\alpha}{x} \\ &= \alpha \cdot \frac{x^\alpha}{x} \\ &= \alpha \cdot x^{\alpha-1} \end{aligned}$$

したがって, $(x^\alpha)' = \alpha x^{\alpha-1}$

問 7

(1) 両辺の自然対数をとると

$$\log y = \log x^{\sin x}$$

$$= \sin x \log x$$

両辺を x で微分すると

$$\frac{d}{dy}(\log y) \frac{dy}{dx} = (\sin x)' \log x + \sin x (\log x)'$$

$$\frac{1}{y} \cdot y' = \cos x \log x + \frac{\sin x}{x}$$

よって

$$\begin{aligned} y' &= y \left(\cos x \log x + \frac{\sin x}{x} \right) \\ &= x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x} \right) \\ &= x^{\sin x - 1} (x \cos x \log x + \sin x) \end{aligned}$$

(2) 両辺の自然対数をとると

$$\log y = \log x^{\frac{1}{x}}$$

$$= \frac{1}{x} \log x$$

両辺を x で微分すると

$$\frac{d}{dy}(\log y) \frac{dy}{dx} = \left(\frac{1}{x} \right)' \log x + \frac{1}{x} (\log x)'$$

$$\frac{1}{y} \cdot y' = -\frac{1}{x^2} \cdot \log x + \frac{1}{x} \cdot \frac{1}{x}$$

$$= \frac{-\log x + 1}{x^2}$$

よって

$$\begin{aligned} y' &= y \cdot \frac{-\log x + 1}{x^2} \\ &= x^{\frac{1}{x}} \cdot \frac{-\log x + 1}{x^2} \\ &= x^{\frac{1}{x}-2}(1 - \log x) \end{aligned}$$

問 8

$$\begin{aligned} (1) \quad y' &= \frac{1}{3x-2} \cdot (3x-2)' \\ &= \frac{3}{3x-2} \end{aligned}$$

$$\begin{aligned} (2) \quad y' &= \frac{1}{\sin x} \cdot (\sin x)' \\ &= \frac{\cos x}{\sin x} \end{aligned}$$

問 9

$$(1) \quad y' = \frac{1}{x \log 10}$$

$$\begin{aligned} (2) \quad y' &= \frac{1}{(x^2+1)\log 2} \cdot (x^2+1)' \\ &= \frac{2x}{(x^2+1)\log 2} \end{aligned}$$

問 10

$$\begin{aligned} (1) \quad y &= \sin^{-1} \frac{\sqrt{3}}{2} \text{ とおくと} \\ \sin y &= \frac{\sqrt{3}}{2} \quad \left(0 < y < \frac{\pi}{2}\right) \text{ であるから} \\ y &= \frac{\pi}{3} \\ \text{よって, } \sin^{-1} \frac{\sqrt{3}}{2} &= \frac{\pi}{3} \end{aligned}$$

$$\begin{aligned} (2) \quad y &= \sin^{-1} \frac{1}{\sqrt{2}} \text{ とおくと} \\ \sin y &= \frac{1}{\sqrt{2}} \quad \left(0 < y < \frac{\pi}{2}\right) \text{ であるから} \\ y &= \frac{\pi}{4} \\ \text{よって, } \sin^{-1} \frac{1}{\sqrt{2}} &= \frac{\pi}{4} \end{aligned}$$

問 11

$$\begin{aligned} AB &= \sqrt{4^2 + 3^2} = 5 \\ \text{よって, } \sin A &= \frac{4}{5} \text{ であるから} \\ A &= \sin^{-1} \frac{4}{5} \\ \text{同様に, } \sin B &= \frac{3}{5} \text{ であるから} \\ B &= \sin^{-1} \frac{3}{5} \end{aligned}$$

問 12

$$\begin{aligned} (1) \quad y &= \cos^{-1} \frac{1}{2} \text{ とおくと} \\ \cos y &= \frac{1}{2} \quad \left(0 < y < \frac{\pi}{2}\right) \text{ であるから} \\ y &= \frac{\pi}{3} \\ \text{よって, } \cos^{-1} \frac{1}{2} &= \frac{\pi}{3} \end{aligned}$$

$$\begin{aligned} (2) \quad y &= \cos^{-1} \frac{\sqrt{3}}{2} \text{ とおくと} \\ \cos y &= \frac{\sqrt{3}}{2} \quad \left(0 < y < \frac{\pi}{2}\right) \text{ であるから} \\ y &= \frac{\pi}{6} \\ \text{よって, } \cos^{-1} \frac{\sqrt{3}}{2} &= \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} (3) \quad y &= \tan^{-1} 1 \text{ とおくと} \\ \tan y &= 1 \quad \left(0 < y < \frac{\pi}{2}\right) \text{ であるから} \\ y &= \frac{\pi}{4} \\ \text{よって, } \tan^{-1} 1 &= \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} (4) \quad y &= \tan^{-1} \sqrt{3} \text{ とおくと} \\ \tan y &= \sqrt{3} \quad \left(0 < y < \frac{\pi}{2}\right) \text{ であるから} \\ y &= \frac{\pi}{3} \\ \text{よって, } \tan^{-1} \sqrt{3} &= \frac{\pi}{3} \end{aligned}$$

問 13

$$\begin{aligned} \text{図より, } \cos y &= \frac{x}{1} = x \text{ であるから} \\ y &= \cos^{-1} x \dots \textcircled{1} \\ \text{また, } \sin y &= \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2} \text{ であるから} \\ y &= \sin^{-1} \sqrt{1-x^2} \dots \textcircled{2} \\ \textcircled{1}, \textcircled{2} \text{ より} \\ \cos^{-1} x &= \sin^{-1} \sqrt{1-x^2} \end{aligned}$$

問 14

$$\begin{aligned} (1) \quad y &= \sin^{-1} \left(-\frac{\sqrt{3}}{2}\right) \text{ とおくと} \\ \sin y &= -\frac{\sqrt{3}}{2} \quad \left(-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right) \text{ であるから} \\ y &= -\frac{\pi}{3} \\ \text{よって, } \sin^{-1} \left(-\frac{\sqrt{3}}{2}\right) &= -\frac{\pi}{3} \end{aligned}$$

$$\begin{aligned} (2) \quad y &= \sin^{-1} \left(-\frac{1}{\sqrt{2}}\right) \text{ とおくと} \\ \sin y &= -\frac{1}{\sqrt{2}} \quad \left(-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right) \text{ であるから} \\ y &= -\frac{\pi}{4} \end{aligned}$$

$$\text{よって, } \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

(3) $y = \cos^{-1} 0$ とおくと

$\cos y = 0$ ($0 \leq y \leq \pi$) であるから

$$y = \frac{\pi}{2}$$

$$\text{よって, } \cos^{-1} 0 = \frac{\pi}{2}$$

問 15

$$\begin{aligned} (1) \quad y' &= -\frac{1}{\sqrt{1-(3x)^2}} \cdot (3x)' \\ &= -\frac{3}{\sqrt{1-9x^2}} \end{aligned}$$

$$\begin{aligned} (2) \quad y' &= \frac{1}{1+\left(\frac{x}{3}\right)^2} \cdot \left(\frac{x}{3}\right)' \\ &= \frac{1}{1+\frac{x^2}{9}} \cdot \frac{1}{3} \\ &= \frac{1}{3+\frac{x^2}{3}} \\ &= \frac{3}{9+x^2} \end{aligned}$$

$$\begin{aligned} (3) \quad y' &= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot (\sqrt{x})' \\ &= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x(1-x)}} \end{aligned}$$

問 16

$$\begin{aligned} y' &= \frac{1}{a} \cdot \frac{1}{1+\left(\frac{x}{a}\right)^2} \cdot \left(\frac{x}{a}\right)' \\ &= \frac{1}{a} \cdot \frac{1}{1+\frac{x^2}{a^2}} \cdot \frac{1}{a} \\ &= \frac{1}{a^2\left(1+\frac{x^2}{a^2}\right)} \\ &= \frac{1}{a^2+x^2} = \frac{1}{x^2+a^2} \end{aligned}$$

問 17

$f(x) = x^3 - x^2 - 2x + 1$ とおくと, $f(x)$ は区間 $[-2, -1]$ で連続である.

また

$$\begin{aligned} f(-2) &= (-2)^3 - (-2)^2 - 2 \cdot (-2) + 1 \\ &= -8 - 4 + 4 + 1 \\ &= -7 < 0 \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^3 - (-1)^2 - 2 \cdot (-1) + 1 \\ &= -1 - 1 + 2 + 1 \\ &= 1 > 0 \end{aligned}$$

よって, 方程式 $f(x) = 0$ は, 区間 $(-2, -1)$ に少なくとも 1 つの実数解をもつ.

問 18

$f(x) = \sin 2x - x$ とおくと, $f(x)$ は区間 $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ で連続である.

また

$$\begin{aligned} f\left(\frac{\pi}{4}\right) &= \sin 2 \cdot \frac{\pi}{4} - \frac{\pi}{4} \\ &= \sin \frac{\pi}{2} - \frac{\pi}{4} \\ &= 1 - \frac{\pi}{4} = \frac{4-\pi}{4} > 0 \end{aligned}$$

$$\begin{aligned} f\left(\frac{\pi}{2}\right) &= \sin 2 \cdot \frac{\pi}{2} - \frac{\pi}{2} \\ &= \sin \pi - \frac{\pi}{2} \\ &= -\frac{\pi}{2} < 0 \end{aligned}$$

よって, 方程式 $f(x) = 0$ すなわち $\sin 2x = x$ は, 区間 $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ に少なくとも 1 つの実数解をもつ.